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Some basic, critical remarks on the reception of the ATD

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On the theory as such

The creation and ongoing development of the anthropological *theory* of the didactic (ATD) demonstrate that, in principle, *this theory cannot be regarded as substitutable* by any of the pre-existing theories (with which the ATD may, of course, converge locally).

Some researchers who claim to operate within the framework of the ATD are tempted to borrow from other ‘theories’ certain praxeological elements $\wp$. On the contrary, each time this question must be considered: How can $\wp$ be modelled in ATD terms? This challenge is one of the countless challenges posed by the effort to ‘think’ the didactic world in ATD terms.

To reiterate, no importation into the ATD of a praxeological element $\wp$ borrowed from another theory can be taken for granted. It must, if necessary, or even if only useful, be modelled and therefore ‘rethought’ within the framework of the ATD itself. Picking up ‘items’ outside the ATD almost always means you are outside the ATD.

Reminder. In the context of the ATD, the word *theory* refers primarily to the component Θ of a praxeology [*T*/τ/θ/Θ]. However, it also refers, as it does in everyday language, to the whole of a praxeological system, in the sense that we speak, for example, of number theory, measure theory, etc. Here the word is understood, by metonymy, in this broad sense.

On the adjective ‘anthropological’

Three concepts are basic to the ATD: those of *institution*, *institutional position* and *person*.

— **Institution.** All human activity takes place within one or more institutions. A family, a school class, a school, a couple, etc. are institutions. *Any institution* can be an object in the field of the ATD.

— **Institutional position.** An institution involves one or more positions. A given institutional position can be occupied or vacant. This is the case in a single-parent family where a mother and her child live alone, with the position of father being vacant. It is also the case in a classroom where the maths teacher is absent: the position remains vacant until a substitute is appointed.

Contrary to certain linguistic habits (where the word ‘teacher’ refers to both the position and the person who occupies it), a position should not be confused with the person who is subjected do it (or could be subjected to it): a teacher or a father is a *person* subjected to some *position* of teacher or father in some institution (a class or a family).

— **Person.** Every human being is subjected to a plurality of institutional positions within a multiplicity of institutions: this is what makes him or her a *person*. As a result, for example, from the point of view of the ATD, a newborn baby *is a person*, notably subjected to the language spoken in the family even if this baby does not yet speak it at all.

Each institutional subjection of a person $\hat{x}$ amounts to subjecting $\hat{x}$ to a specific system of *conditions and constraints*, which (partially) determines $\hat{x}$'s behaviour. This system of conditions and constraints is generated, at a given moment, by the set of institutional positions and the persons who occupy them.

If $\hat{x}$ is a student, these conditions and constraints will be generated in particular by the positions of teacher, mother and father, older brother and sister. The list of relevant positions cannot be established once and for all: for a given question, didacticians must endeavour to identify them—without, as a general rule, being sure that they are exhaustive.

Compared to other theories of human learning, the ATD is not limited to a system of positions considered a priori to be ‘specific’ to the question under study. In many cases, for example, it is necessary to take into account the position of teacher educator, a position that belongs to the noosphere of the institution in question.

Other positions are less studied. To cite one example, this is the case with the so-called *euergetic position*, the position of *euergetes*, i.e. ‘benefactors’ of teachers and the teaching they provide. In the case of mathematics, we thus see mathematicians trying to help teachers in the name of their knowledge of mathematics, and not because they are knowledgeable in didactics.

Researchers in didactics ξ are 'defined' by their position p_ξ. The same person $\hat{x}$ may occupy the position p_ξ and at other times the teacher's position $\hat{y}$. Their cognitive equipment does not, as a rule, concern the same objects. In the case of 'identical' objects (e.g., the 'student'), the 'useful' relation of $\hat{x}$ to such an object differs depending on whether it is mobilised in position $\hat{y}$ or position p_ξ.

Researchers ξ must never believe that they can magically replace the teacher. The reverse is equally true, of course. This should in no way be accompanied by ξ's patronizing attitude towards teachers (or vice versa): in many cases, as everyone knows, they are often the very same persons…

On cognition and 'the didactic'

The term 'instance' refers either to a person $\hat{x}$ or to an institutional position $\hat{p}$. An essential concept in the ATD is that of the relation of an instance $\hat{\imath}$ to an object σ, denoted by $R(\hat{\imath}, \sigma)$, or $R(\hat{x}, \sigma)$ if $\hat{\imath}$ is the person $\hat{x}$ and $R(\hat{p}, \sigma)$ if $\hat{\imath}$ is the institutional position $\hat{p}$. The relation $R(\hat{\imath}, \sigma)$ is the complex of praxeologies used by $\hat{\imath}$ that involve the object σ.

An instance $\hat{\imath}$ *does not know* the object σ if $R(\hat{\imath}, \sigma) = \emptyset$ and *knows* σ if $R(\hat{\imath}, \sigma) \neq \emptyset$. In a set $\bar{I}$ of institutions, an object σ is an entity that exists for at least one instance $\hat{\imath}$. It should be emphasised that, in the ATD, the verb 'to know' *does not have a single meaning*. One can know an object σ simply by having 'heard about' σ. It is incumbent upon the researcher ξ to study the relation $R(\hat{\imath}, \sigma)$.

A ‘gesture’ δ performed by an instance $\hat{u}$ may modify the relation of some instances $\hat{\imath}$ to a number of objects σ. For an instance $\hat{w}$ to regard δ as *didactic in the broad sense* in relation to (σ, $\hat{\imath}$), $\hat{w}$ must judge that δ will be regarded by a certain evaluating instance $\hat{v}$ as likely to modify the proximity of $R(\hat{\imath}, \sigma)$ with a certain relation $R(\hat{s}, \sigma)$ considered by $\hat{v}$ to be the ‘correct’ relation to σ.

If $\hat{w}$ judges that $\hat{v}$ will judge that this proximity will increase, i.e. that $\hat{\imath}$ will learn about σ, δ will be judged by $\hat{w}$ as *didactic in the strict sense*; if $\hat{w}$ judges that $\hat{v}$ will judge that this proximity will decrease, δ will be judged by $\hat{w}$ as *anti-didactic*. If $\hat{w}$ judges that $\hat{v}$ will judge that the proximity between $R(\hat{\imath}, \sigma)$ and $R(\hat{s}, \sigma)$ will not essentially change, δ will be judged by $\hat{w}$ as *isodidactic*.

Relative to a system $(\hat{\imath}, \sigma, \hat{s}, \hat{v}, \hat{w})$, *the didactic* is the set of pairs $(\hat{u}, \delta)$ such that δ is judged by $\hat{w}$, referring to $\hat{v}$ and $\hat{s}$, as didactic in the broad sense relative to $R(\hat{\imath}, \sigma)$. A gesture δ held by $(\hat{s}, \hat{v}, \hat{w})$ to be anti-didactic or isodidactic relative to $(\hat{\imath}, \sigma)$ can be regarded *at the same time*, according to $(\hat{s}, \hat{v}, \hat{w})$, as didactic in the strict sense relative to another pair $(\hat{\imath}', \sigma')$, e.g. when $\hat{\imath}' = \hat{\imath}$ and $\sigma' \neq \sigma$.

A classic case in standard teaching is as follows: $\hat{\imath}$ is the position of student $\hat{x}$ and $\hat{u}$ is the position of teacher $\hat{y}$, with the object σ being an object 'to be taught' for $\hat{y}$ and an object 'to be learned' for $\hat{x}$; furthermore, we have $\hat{w} = \hat{v} = \hat{s} = \hat{y}$, the action δ performed by $\hat{u} = \hat{y}$ being judged by $\hat{w} = \hat{v} = \hat{y}$ as didactic in the strict sense (or, at a pinch, as isodidactic).

In such a case, the action δ may be *intentional* (on the part of $\hat{u} = \hat{y}$) *or not*. In all cases, δ contributes to the system of conditions and constraints of the study of σ by $\hat{x}$. When, in the above, we take $\hat{u} = \hat{x}$ and $\hat{x} = \hat{w} = \hat{v}$, with, as always, $\hat{s} = \hat{y}$, we speak of a *self-study* gesture. This assumes that the relation of $\hat{x}$ to the relation of $\hat{y}$ to σ, i.e. $R(\hat{x}, R(\hat{y}, \sigma))$, is non-empty.

Didactics has *the didactic* as its object: it studies the gestures δ that can be performed by various instances $\hat{u}$ regarded by an instance $\hat{w}$ as likely to generate systems of conditions and constraints under which a specific relation $R(\hat{\imath}, \sigma)$ of an instance $\hat{\imath}$ to an object σ is judged by a certain instance $\hat{v}$ to be closer to a reference relation $\check{R}(\hat{s}, \sigma)$.

The *science of the didactic* is the specific contribution of didactics to our knowledge of institutional and personal worlds. This project of scientific elucidation faces a formidable obstacle almost everywhere: the *denial*, if not utter dismissal, of the didactic (when it is perceived), correlative to the promotion and valorisation of knowledge.

The promotion of knowledge promotes inattention to *non-knowledge*—except among a few people, who adopt the implicative schema *Knowledge* ↩ *Learning* ↩ *Studying*. But the dominant pattern is often this: *Knowledge* ↩ ∅. In a letter (1737), Voltaire wrote: ‘I take great pains to spare our Frenchmen, who, generally speaking, would like to learn without studying.’

Before him, in *Les précieuses ridicules* ('The pretentious young ladies', 1659), Molière gives the valet Mascarille (originally played by Molière himself) a reflection worthy of a *sociology of the didactic*: 'People of rank know everything without ever having learned anything.' The observation here is one of denial of the didactic.

In contrast, there is a ‘didactic elite’ that places at the forefront *non-knowledge* and the *study* that allows to overcome it. In his *Discourse on the Method* (1637), Descartes wrote: ‘… the little I have hitherto learned is almost nothing in comparison with that of which I am ignorant, and to the knowledge of which I do not despair of being able to attain…’

For the enthusiasts of learning, *any* social situation can be didactic. In his *Journal d'un poète* (1883-1885), Alfred de Vigny wrote: 'I have never met a man from whom there was nothing to learn.' Before him, Galileo Galilei had said about the same: *Numquam inveni hominem tam ignarum ut non possem aliquid ab eo discere*. (See Google translate.)

In general, the didactician must be attentive to all institutional or personal actions that have didactic potential for a given instance $\hat{\imath}$ in relation to a given object o, particularly when $\hat{u} = \hat{\imath}$, i.e. in the case of potentially self-didactic gestures, which are too often neglected in favour of large structures with didactic intent (courses, practical work, etc.).

At the same time, attention should be paid to the conditions and constraints that prevent a potentially didactic gesture from becoming didactic in the strict sense for some instance $\hat{w}$. This is the case, for example, with the lack of valorisation by $\hat{w}$ (with $\hat{w} = \hat{u}$, or $\hat{w} = \hat{\imath}$, etc., as applicable), *not* of the object $\mathcal{O}$ itself, but of the *knowledge* of the object $\mathcal{O}$, and therefore of its study.

This is also the case if $\hat{w}$ has a generally *adultist* attitude, i.e. an epistemic relation to the world subject to the tyranny of one’s supposed ‘opinion’, far removed from the essential self-didactic capacity to ‘suspend one’s judgement’, in agreement with the old Greek concept of *epoché* (ἐποχή), sometimes expressed as one’s *withholding of assent*.

On models and modelling

Saussurean linguistics distinguishes between the *signifier*, the *signified* and the *referent*. One of the obstacles to the reception of a theory—and this is true of the ATD—is that many instances identify the signified (e.g., the concept of student) of a signifier (the word ‘student’) with what they consider to be a pre-established referent (*my* students).

This is why what we called in France *élèves professeurs* (‘pupil teachers’, i.e. student teachers) protested against this designation on the grounds that ‘they were no longer *élèves*’! The faithful reception of the ATD requires that *î* re-examine the system of referents to which certain signifiers refer and restore the semantic fullness of the signified.

This applies fully to the signifier ‘model’. A common misconception is that we only talk about a model $\mathscr{M}$ of a system $\mathcal{S}$, where $\mathscr{M}$ is a ‘mathematical’ model, if $\mathcal{S}$ is regarded as a ‘non-mathematical’ system. This is a limitation that is foreign to the ATD: a model $\mathscr{M}$ of a system $\mathcal{S}$ is itself a system $\mathcal{S}'$ and any system can, a priori, be a model of another system.

It should be emphasised that a system is only known through a model of that system, which is a supposed reality conceptualised using the model in question. Any object σ can be regarded as a system $\mathcal{S}$. In his book *Creative Evolution*, published in 1907, Henri Bergson wrote: ‘Between thinking an object and thinking it existent, there is absolutely no difference.’

Any object $\mathcal{o}$ existing for an instance $\hat{w}$, i.e. any system $\mathcal{S}$, is assumed by $\hat{w}$ to be subject to its *own laws*, which $\hat{w}$ does not necessarily know, but may wish to know. As an example, consider the case where $\hat{w}$ is the mathematician's position and where $\mathcal{o} = \mathcal{S} = \sqrt{2}$. The ostensive complex $\sqrt{2}$ refers to a positive number whose square is equal to 2, that is such that $(\sqrt{2})^2 = 2$.

This system $\mathcal{S}$ has many other models, for example the ostensive complex $2^{½}$. An important model is simply the equation $x^2 = 2$ for $x > 0$, from which we can derive this other model: $x > 0$ and $x = \dfrac{2}{x}$. One key law governing this model is as follows: if $\dfrac{2}{x'} > x'$, then $x' < \sqrt{2}$.

Since $2/1.414 = 1.4144... > 1.414$, we conclude that $1.414 < \sqrt{2}$. This technique allows us to obtain – painstakingly, of course – the successive decimal places of $\sqrt{2}$. Thus, we have $2/1.4141 = 1.4143... > 1.4141$, $2/1.4142 = 1.41422 > 1.4142$, $2/1.4143 = 1.4141... < 1.4143$: we conclude that $\sqrt{2} = 1.4142...$ Similarly, we will obtain $\sqrt{2} = 1.41421...$, etc.

In the previous situation, we used a model of $\sqrt{2}$, namely $x^2 - 2 = 0$, with $x > 0$. Other models of a similar nature are possible. For example, for $x > 0$, we have: $x^2 = 2 \Rightarrow x^2 - 1 = 1 \Rightarrow (x - 1)(x + 1) = 1 \Rightarrow x - 1 = \dfrac{1}{x+1} \Rightarrow x = 1 + \dfrac{1}{x+1}$. This equation is, for $x > 0$, another model of $\sqrt{2}$.

The function f defined for $x > 0$ by $f(x) = 1 + \frac{1}{x+1}$ has a fixed point, which is none other than $\sqrt{2}$: $f(\sqrt{2}) = 1 + \frac{1}{\sqrt{2}+1} = 1 + \sqrt{2} - 1 = \sqrt{2}$. This function is strictly decreasing on $[0, +\infty[$, so that $f([1.414; 1.415]) = [f(1.415); f(1.414)] = [1.41407867...; 1.4142502071...] \subset [1.414; 1.415]$.

We can define a sequence $(u_n)_{n \geq 0}$ by $u_0 = 1.414$ and, for all $n \geq 1$, $u_n = f(u_{n-1})$. The function f is *contracting* on $I = [1.414; 1.415]$: if x, $y \in I$, then $|f(x) - f(y)| = \dfrac{1}{(x+1)(x+1)} |x - y| < \dfrac{1}{2.414^2} |x - y| < 0.172\ |x - y|$. So that: $|u_n - \sqrt{2}| = |f(u_{n-1}) - f(\sqrt{2})| < 0.172\ |u_{n-1} - \sqrt{2}| < 0.172^2\ |u_{n-2} - \sqrt{2}| < \ldots < 0.172^n\ |u_0 - \sqrt{2}| < 0.172^n \times 0.0003$.

For $n = 9$, we have: $|u_9 - \sqrt{2}| < 0.00000000000395254912990715314176$. The Excel spreadsheet displays the following values: $u_1 =_{xl} 1.414250207$; $u_2 =_{xl} 1.414207275$; $u_3 =_{xl} 1.414214641$; …; $u_9 =_{xl} 1.414213562$. A powerful calculator directly gives, for example, this result:

$$\sqrt{2} = 1.41421356237309504880168 87...$$

This technique for working with models, which moves from the model $x^2 = 2$ to the model $x = 1 + \frac{1}{x+1}$ and then to the model $f(x) = 1 + \frac{1}{x+1}$—a process which illustrates the *recursivity* of modelling—can be generalised to systems of the form $\sqrt{\alpha}$ where α is not a perfect square.

Thus, if $\alpha^2 = 17$, we will have the models $x^2 = 17$, then $x = 4 + \dfrac{1}{x+4}$ and therefore $f(x) = 4 + \dfrac{1}{x+4}$. Here, starting from $x_0 = 4$, we will have $u_8 =_{\text{xl}}$ 4.12310562561766. With the calculator already used we get

$$\sqrt{17} = 4{,}12310562561766054982\ldots$$

In all of the above cases, we start with a system $\mathcal{S}$ about which we ask a certain question $\mathcal{Q}$. In order to answer $\mathcal{Q}$, we consider a model $\mathcal{M}$ of $\mathcal{S}$ that we try to 'make speak'. But this relationship *can be reversed*: if $\mathcal{M}$ is a model of $\mathcal{S}$, then $\mathcal{S}$ can be regarded as a model of $\mathcal{M}$. The modelling relationship is *recursive*; we will see now that the modelling relationship is also *reversible*.

A right-angled triangle with sides 2 and 5 has a hypotenuse of length $\sqrt{2^2 + 5^2} = \sqrt{29}$: the system $\sqrt{29}$ is thus a model of this triangle. Conversely, this triangle is a *geometric* model $\mathscr{M}_0$ of $\sqrt{29}$. In its turn, this model $\mathscr{M}_0$ can be modelled by a *graphical* model $\mathscr{M}_1$, a right-angled triangle drawn on paper with sides 2 and 5: an approximate value for $\sqrt{29}$ is obtained by measuring its hypotenuse.

In a book published in 1912 in the US, the German mathematician Carl Runge (1856–1927) proposed this figure for calculating $x = \frac{a}{b} \times c$:

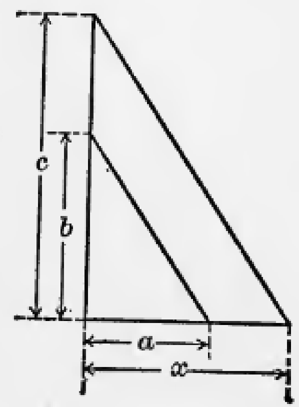

FIG. 3.

As is always the case in graphical calculation, the proper modelling of the expression $\frac{a}{b} \times c$ first requires a *geometric* model based on provable theorems, followed by a *graphical* model that will yield an approximate numerical value for $\frac{a}{b} \times c$.

Graphical calculation techniques were used for around a century, until the 1970s, by engineers and architects. This was the method used, for example, by the Catalan architect Antoni Gaudí (1852–1926), who designed Barcelona's *Sagrada Família*. (When Gaudí died, less than a quarter of the project had been completed.)

The common notion of a model of a *non*-mathematical system extends to the case of mathematical modelling of *mathematical* systems. *Mathematical activity consists of defining and studying such models*, as shown (above) in the case of contracting functions and fixed-point theorems (see for example at https://en.wikipedia.org/wiki/Fixed-point_theorem).

The purpose of studying a model $\mathscr{M}$ is to make the system that is this model 'speak' in order to gain knowledge about the system $\mathcal{S}$ that we are seeking to 'better understand'. It is for this purpose that, *in principle*, the mathematical works that make up traditional curriculums *are created and should be used*.

On questioning the world

Traditional teaching refers to lists of works (mathematical, physical, etc.) to be studied and which the teacher must ‘teach’ to students. This immerses us in the paradigm of visiting works. In mathematics, we visit natural numbers, fractions, decimal numbers, etc., with a guide who, in this case, is the ‘mathematics’ teacher.

The ATD does not reject the study of works that are *tools for studying questions*. However, through the concept of *questioning the world*, it aims to redefine its motives and content almost radically. Here is an example. In order to make the most of the graphical technique based on Pythagoras' theorem seen above, an instance $\hat{\imath}$ inquires into the following question *Q*:

'Which natural numbers can be written as the sum of two integer squares?' In other words: 'Which integers $\alpha > 0$ can be modelled in the form $\alpha = a^2 + b^2$, with $a, b \in \mathbb{N}$?' To answer *Q* one has to break down the number into prime factors and then use a theorem that took a long time to establish (see, e.g., at https://en.wikipedia.org/wiki/Fermat's_theorem_on_sums_of_two_squares):

‘An integer greater than or equal to 1 is the sum of two squares if and only if each of its prime factors of the form $4k + 3$ occurs with an even exponent.’ For example, 32 × 53 = 1125 is the sum of two squares (we have 1125 = 900 + 225 = $30^2 + 15^2$), but 3 × 625 = 1875 is not. More generally, α, modelled by its decimal notation $\mathscr{M}_0$, is then modelled by its prime factorisation $\mathscr{M}_1$.

For example, $\alpha = 187$ ($\mathscr{M}_0$) can also be written as 11×17 ($\mathscr{M}_1$). We have $17 = 16 + 1 \equiv 1 \pmod 4$ but $11 = 8 + 3 \equiv 3 \pmod 4$: since the exponent of 11 is odd, $\alpha = 187$ is *not* the sum of two squares. On the other hand, $\beta = 2057 = 11^2 \times 17$ can be written as $11^2 + 44^2$ ($\mathscr{M}_2$).

The ATD provides a general analysis of the inquiry conducted into a given question Q. This inquiry is modelled using what is known as the *Herbartian schema*, the semi-developed form of which is as follows:

$$[S(\hat{X}, \hat{Y}, Q) \Rightarrow M] \Rightarrow A^{\heartsuit}.$$

Here, $\hat{X}$ is the team of inquirers (researchers, students, etc.) and $\hat{Y}$ is the team of 'supervisors' (teachers, etc.).

$A^{♥}$ is the answer—always provisional, often partial, sometimes lacking in solidity—to which the didactic system $S(\hat{X}, \hat{Y}, Q)$, which can also be called an 'inquiry system' or 'study and research system', will have arrived when the decision is made to stop—provisionally?—the inquiry into Q.

The letter *M* refers to the so-called *didactic milieu*, which can also be called the *milieu for the inquiry* (into *Q*) or for the *study and research* (relative to *Q*). This resource system is built up by the system $S(\hat{X}, \hat{Y}, Q)$ during the inquiry. Relative to the question studied above the inquiry outlined thus introduced into *M* Fermat's two-square theorem and the prime factorisation of an integer.

The composition of the milieu M is generally represented by the formula

$$M = \{A_1^\Diamond, A_2^\Diamond, \ldots, A_n^\Diamond, Q_1, \ldots, Q_m, W_1, \ldots, W_p\}.$$

The resources denoted $A_i^\Diamond$ are answers to the question Q that exist in various institutions. In the example given, the answer $A_i^\Diamond$ chosen is unique and may constitute the answer $A^\heartsuit$ validated by the inquiry system $S(\hat{X}, \hat{Y}, Q)$ at the end of its enquiry.

Of course, this answer will most likely generate this question Q_j, which we will denote here as Q_π: 'How can we obtain the prime factorisation of an integer α?' The answer chosen depends on the type of institutions in which $S(\hat{X}, \hat{Y}, Q)$ operates. In a 'scholarly' institution, the inquiry could use the study at https://en.wikipedia.org/wiki/Integer_factorization.

On the other hand, in an institution with ‘mere’ users, an online calculator can be integrated into the milieu to factorise the numbers of interest. An online calculator may as well be used to find the decomposition, where it exists, of the number in question into the sum of two squares.

$$n = 2{,}057$$
$$= 11^2 \cdot 17^1$$

	Sums of two squares	Multiplied out
1	$11^2 + 44^2$	$121 + 1{,}936$

The difficulty in introducing the paradigm of questioning the world into teaching institutions is the effect of various conditions and constraints. What often appears to be the case is that the ‘study and research paths’ (SRP) developed by ATD researchers are ‘haunted’ by the visit of (‘prestigious’) works.

In the example developed above, thus, mathematicians specialising in didactics may see above all an opportunity to encounter, study, and even ‘prove’ Fermat’s theorem on the sum of two squares, rather than identifying the limitations of a certain technique for graphically calculating an integer square root—by measuring the hypotenuse of an appropriate right-angled triangle.

This surreptitious return of the paradigm of visiting works to the very heart of the paradigm of questioning the world calls for an in-depth causal analysis. The ‘shift’ generated by the introduction of the paradigm of questioning the world leads especially to the enrichment of the validation praxeologies that have become standard in mathematics teaching.

A very simple example can illustrate this phenomenon. Consider the following question: 'What is the solution to the equation $3x - 7 = 2$?' A certain mathematics teacher expects his young students to perform the following calculation:

$$3x - 7 = 2 \Leftrightarrow 3x = 2 + 7$$
$$\Leftrightarrow 3x = 9 \Leftrightarrow x = 9/3 \Leftrightarrow x = 3.$$

A student suggests checking the validity of the value found ($x = 3$) by substituting this value into the given equation: he finds that $3 \times 3 - 7 = 9 - 7 = 2$, as expected. Instead of praising him, the teacher protests: this suggestion is ‘useless’, he comments, since we proceeded ‘by equivalences’—which is true.

But it is not clear that the above calculation ‘by equivalences’ is clearly understood (at least that is what the student’s suggestion suggests) as being the conjunction of the two sequences of implications: $3x - 7 = 2 \Rightarrow 3x = 2 + 7 \Rightarrow 3x = 9 \Rightarrow x = 9/3 \Rightarrow x = 3$ *and*

$x = 3 \Rightarrow 3x = 9 \Rightarrow 3x = 2 + 7 \Rightarrow 3x - 7 = 2$.

(This ‘reverse’ sequence may seem strange to the casual reader…)

The validation enabled by the free exercise of the *dialectic of media and milieus* thus requires both $\hat{X}$ and $\hat{Y}$ to accept a certain *redundancy* in the apparatus of ‘proofs’, contrary to the minimalist ideal of a certain Jansenistic mathematical culture. However this does not mean replacing a minimum amount of evidence with a maximum amount.

It means resorting to an *optimum* of evidence *with respect to* $S(\hat{X}, \hat{Y}, Q)$, allowing a ‘robust’ relation to the objects in question to be constructed. In the example above, we could humbly include the following calculation in such an optimum of proofs, which illustrates *another technique* for solving the equation: $3x - 7 = 2 \Rightarrow x - 7/3 = 2/3 \Rightarrow x = 2/3 + 7/3 = 9/3 = 3$.

The search for minimal evidence generally constitutes an *epistemological and didactic obstacle*, including in mathematics. This is even more so when the question Q to be studied only partially belongs to the mathematical universe, but also to other fields of knowledge (physics, literature, history, biology, etc.).

In this case, didacticians of mathematics may be less fascinated by ‘illustrious’ works (which they know little or nothing about) than by what they see as their ‘incompetence’, trapped as they are in the *retrocognitive* attitude engendered by their subjection to the paradigm of visiting works—instead of developing the *procognitive* attitude inherent in questioning the world.

To put it succinctly, it is by inquiring into a *multitude of questions* that one becomes ‘knowledgeable’. Raising a question is first and foremost a potentially *didactic* gesture, that enables learning. Of course, all this applies when we replace ‘didactician of mathematics’ with ‘didactician of history’, or ‘didactician of literature’, or ‘didactician of biology’, etc.

Another obstacle to the advent of the paradigm of questioning the world is the attitude towards the objects o which the question Q involves. Education in a discipline $\mathcal{D}$ tends to develop an 'epistemophilic' attitude towards objects o 'within the discipline' and, too often, an 'epistemophobic' attitude towards objects outside the discipline $\mathcal{D}$.

These objects will be regarded, for example, as ‘ill-defined’, ‘ideological’, ‘incomprehensible’, and unworthy of consideration. The basic work of the didactician ξ consists, in general, of studying the systems of conditions and constraints that have generated or are likely to generate a given relation $R(\hat{\imath}, o)$, for any object o that exists for at least one person or institution.

This should be the case even if, initially, σ did not exist for ξ. Attitudes that can be described as ‘objectophile’ and ‘objectophobic’ are epistemological obstacles for ξ. Given the current division into disciplines $\mathcal{D}_\ell$, overcoming the paradigm of visiting works will require replacing a curriculum listing works from $\mathcal{D}_\ell$ with one composed of *questions*.

The study of such questions is supposed to involve works relating (at least partially) to the main discipline studied. As an example, here is one such ‘easy’ question: ‘We sometimes hear that, in order for a population not to age, the average number of children per woman should be at least 2.1. What exactly does this mean and how is the value 2.1 calculated?’

More generally, I believe that all teaching should contain a potentially didactic structure, which I have called a *forum for questions*, where the questions to be studied would be discussed. Here is a (nonsensical?) question which could be considered *or not*: ‘Today is called *Blue Monday*, which is held to be the most depressing day of the year. Where does that come from?’

In truth, the chosen question Q, however naive it may be, must be 'interesting' for $\hat{X}$ in terms of their knowledge of the world and must appear manageable by $\hat{X} \cup \hat{Y}$, provided $\hat{X}$ and $\hat{Y}$ adopt a suitable *procognitive* attitude. *Objectal* diversity and *didactic* diversity are at the heart of the researcher ξ's work. That, it seems to me, is where we are now.

As this presentation concludes, I would like to extend my special thanks to Yves Matheron, who cannot be present at the CRM today, but who provided me with invaluable assistance in preparing the above. Of course, any weaknesses in my presentation cannot be attributed to him in any way.

Thank you for listening, folks!