Study and Research Paths and questioning the world – the development of research practices in the ATD

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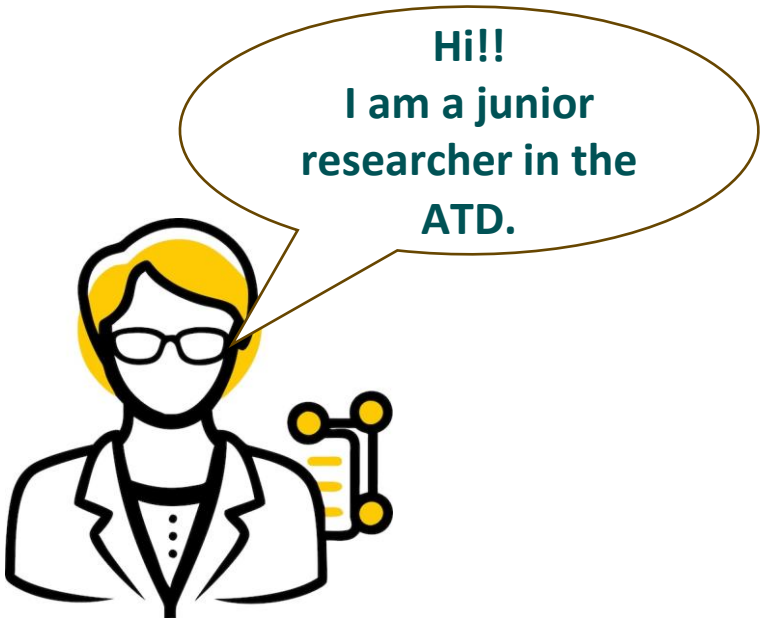
Overview of the talk

- The emergence of Study and Research Paths (SRPs): results of the ATD research
- Getting to know SRP
- Didactical design of SRP
- SRP as an object of study



I am going to share my study process on SRP research.

Overview of the talk

- The emergence of Study and Research Paths (SRPs): results of the ATD research
- Getting to know SRP
- Didactical design of SRP
- SRP as an object of study

Personal routes are different, resources many

Moment of first encounter (or re-encounter)

STEPS TOWARDS A NEW EPISTEMOLOGY IN MATHEMATICS EDUCATION

Yves Chevallard, *IUFM d'Aix-Marseille, France*

…act. *This talk will centre on a number of inadequacies that beset mathematics education in present-day societies. In the author's view, the basic obstacles lie in – or can be expressed in terms of – the epistemological "regime" of the knowledge imparted by most scholastic institutions, as well as in its main social, cultural, and political correlates. It is therefore an essential responsibility – though not a monopoly! – of researchers in didactics to contribute to the advent of a new school epistemology, more in tune with the needs of our time – a crucial pursuit on which this presentation simply aims to shed some light.*

L'ANALYSE DES PRATIQUES ENSEIGNANTES EN THÉORIE ANTHROPOLOGIQUE DU DIDACTIQUE

Yves Chevallard*

ABSTRACT

This paper originated in a series of lectures given at the summer university for in-service mathematics teachers held in La Rochelle (France) in july 1998. It provides a straightforward presentation of some key concepts of the anthropo-logical approach to the teaching of mathematics. It is mainly intended to help the practising didactician come into closer contact with the major implications

USING *RESEARCH AND STUDY COURSES* FOR TEACHING MATHEMATICAL MODELLING AT UNIVERSITY LEVEL

Berta Barquero*, Marianna Bosch**, Josep Gascón*

*Universitat Autònoma de Barcelona, **Universitat Ramon Llull (Barcelona)

This paper focuses on the teaching of mathematical modelling during the first year of experimental sciences university degrees. Within the frame of the Anthropological Theory of the Didactic (ATD), we propose the design of Research and Study Courses (RSC) as a new didactic device to teach mathematical modelling with a double purpose: to make students aware of the rationale of the mathematical contents they have to learn and to connect these contents through the study of open modelling questions. We also show to what extent these courses can "cover" the considered mathematical curricula giving a clear functionality to its different contents.

Afterthoughts on a Seeming Didactic Paradox

Yves Chevallard

UMR P3 ADEF, INRP, Aix-Marseille Université

The first part of this contribution consists in an off-the-cuff reaction to the presentation by Judith S. Lederman, Norman G. Lederman, and Per-Olof Wickman of their study of inquiry-based *versus* direct instruction. The second part was written some time later. It aims to open up a passage allowing educational research relating to the "inquiry" paradigm to free itself from the dictur

Vers une didactique de la codisciplinarité
Notes sur une nouvelle épistémologie scolaire

par Yves Chevallard
IUFM d'Aix-Marseille & UMR ADEF

Résumé. – Une foule de praxéologies scolaires émergentes – tel, en France, en fin de lycée, le dispositif des *travaux personnels encadrés* –, encore fragiles prises une à une sans doute, mais très insistantes à travers leur principe actif commun, ébauchent peu à peu un nouveau paradigme scolaire qui pourrait être la matrice d'une épistémologie plus authentique, en rupture de deux façons indissociables avec l'épistémologie scolaire dominante : d'une part, parce qu'elle induit une mobilisation (ou une construction) *fonctionnelle* de connaissances et de savoirs, en vue de répondre à une question motivée, qui délimite localement le champ de l'étude ; d'autre part, parce que les connaissances disciplinaires qu'une telle étude mobilise ou porte à construire interviennent alors de façon *codisciplinaire*, c'est-à-dire dans une synergie nécessaire avec des connaissances relevant d'une pluralité de juridictions disciplinaires, dont la question étudiée et son étude déterminent l'élection. Si la première règle est familière aux didacticiens (sinon au monde scolaire qu'ils étudient), la seconde l'est beaucoup moins, parce que la science didactique a repris en écho un trait vécu comme définitoire du monde scolaire (plutôt que savant) : la fragmentation, l'enfermement, voire l'agressivité disciplinaire sur lesquels ce monde s'est bâti. Une didactique de la nouvelle épistémologie scolaire est ainsi en grande partie à penser.

A bridge between inquiry and transmission: the study and research paths at university level

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This paper focuses on the notion of 'Study and Research Path' (SRP) proposed in the frame of the Anthropological Theory of the Didactic, as it was designed and implemented in first-year courses at university level of business and administration degrees. First, we show how SRPs can 'live' at university level, describing the conditions and constraints under which they take place in two different university institutions. Secondly, we focus on how SRPs can promote the interaction between different teaching approaches: those derived from inquiry-based models and those based on transmission pedagogies. Finally, we indicate why this interaction is essential as it enriches the students' milieu and enables the dialectics of questions and answers, both crucial elements in the evolution of the study process.

The ecology of study and research paths

This module develops the analysis in terms of inquiry process to approach the critical issue of the conditions needed for the paradigm of questioning the world to prevail in our educational systems.

Presentation by Marianna Bosch Casabò

IQS School of Management, Univ. Ramon Llull, Barcelona

https://www.mathunion.org/icmi/awards/amor/yves-chevallard-unit

The emergence of Study and Research Paths (SRPs)

Results from ATD research

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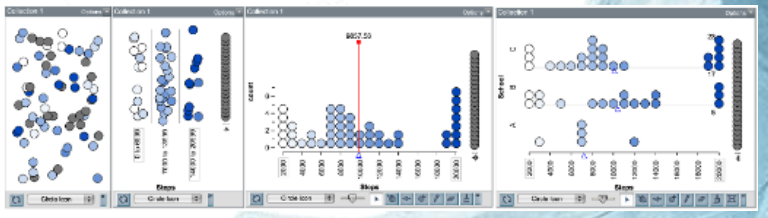
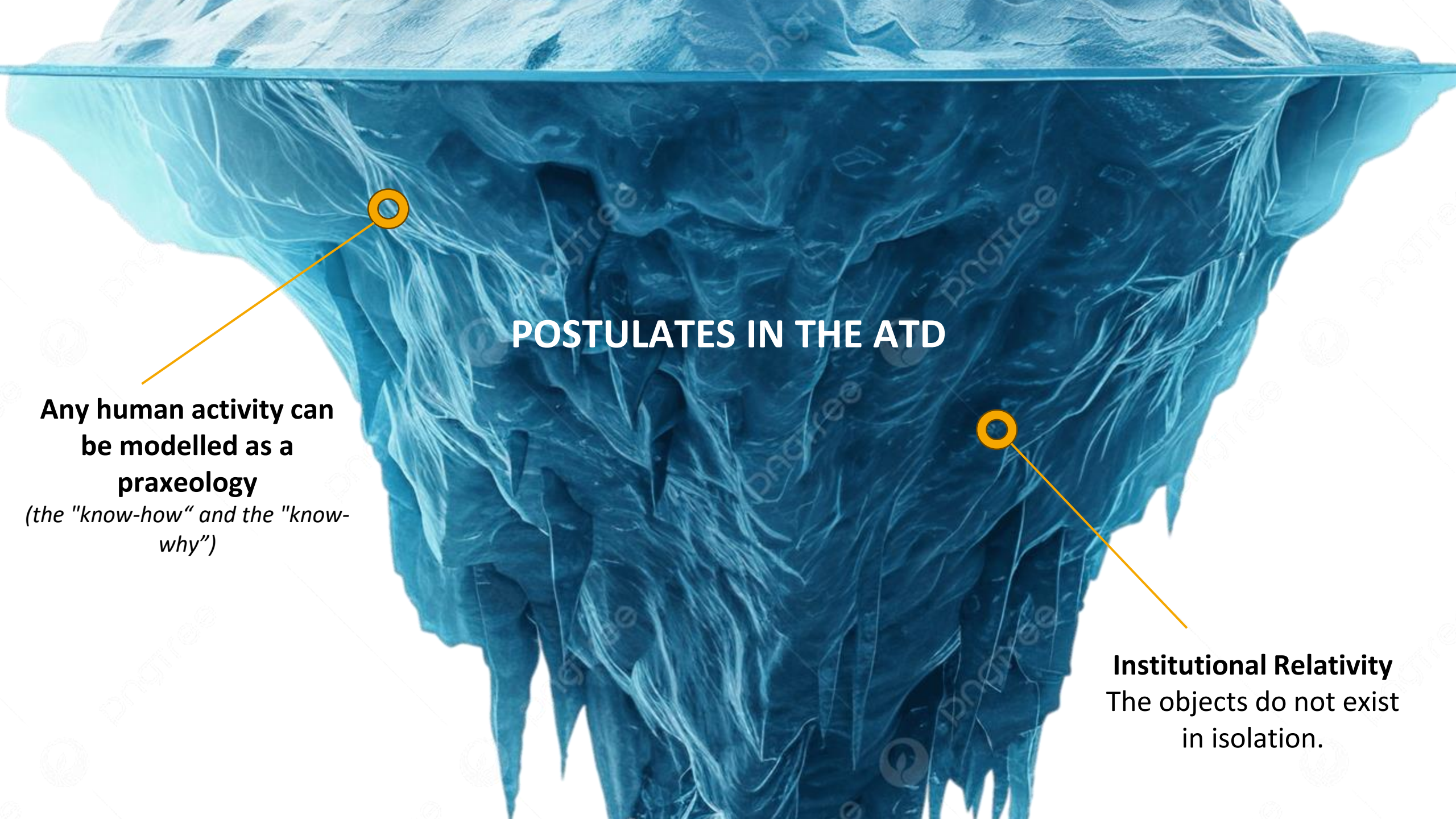
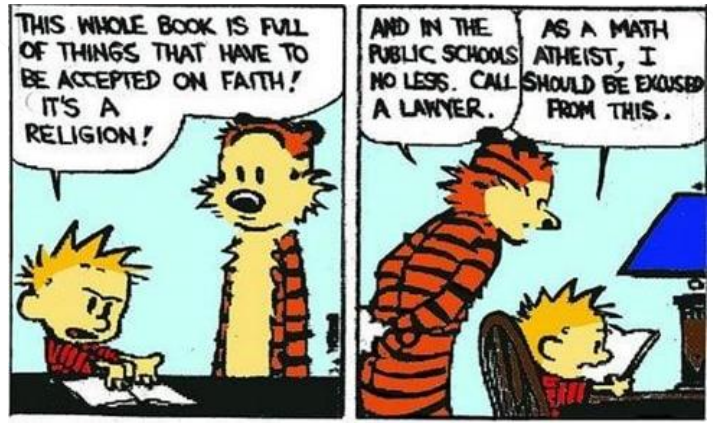
The emergence of SRP



ONLY A METHODOLOGY FOR THE INQUIRY PROCESS?


POSTULATES IN THE ATD
Any human activity can be modelled as a praxeology
(the "know-how“ and the "know-why”)
Institutional Relativity
The objects do not exist in isolation.



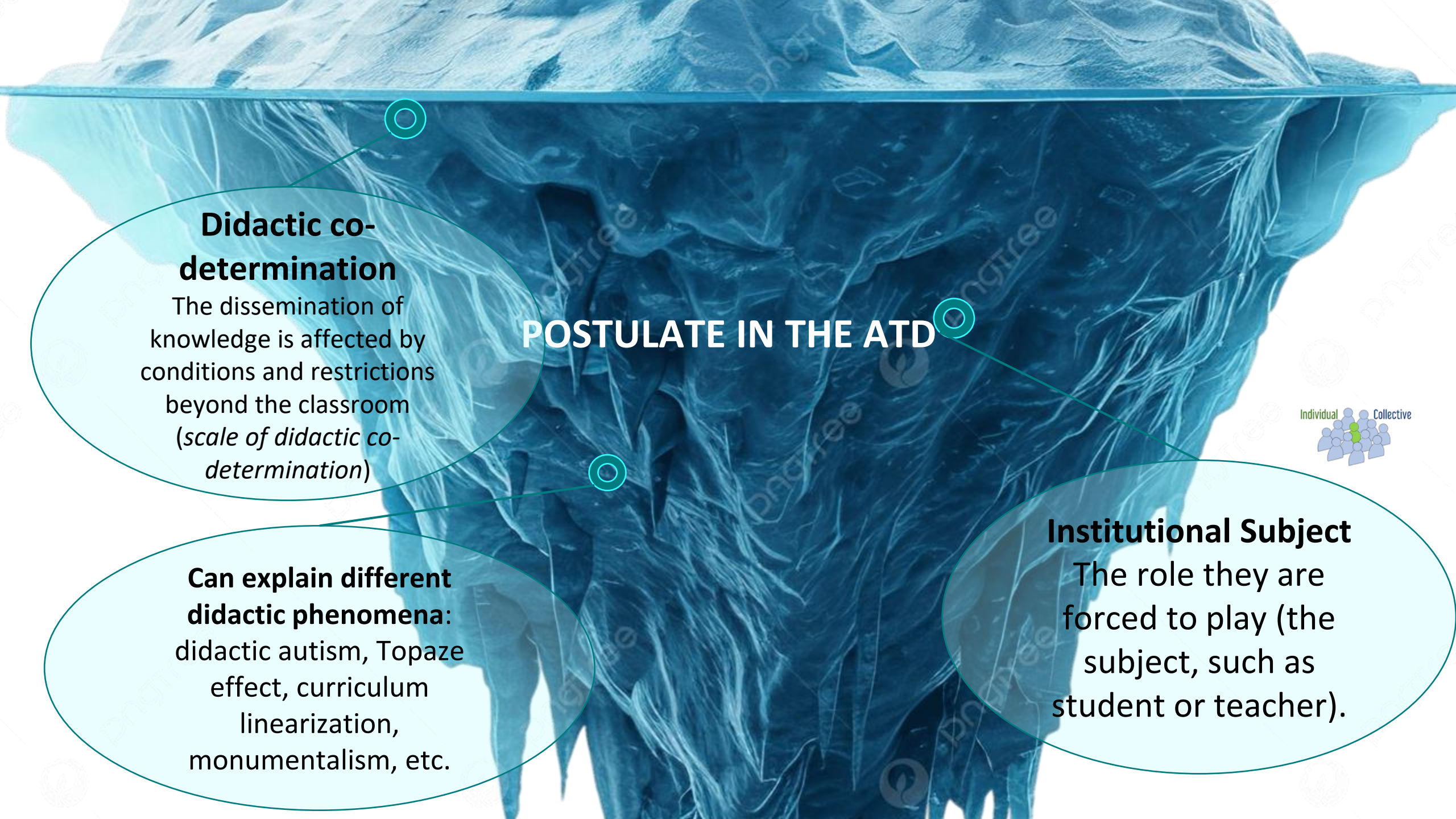
If mathematics is a religion, then I am not a believer…

The Postulate of Didactic Transposition.

Knowledge undergoes a series of adaptive transformations to become teachable within specific institutions

→ Didactic phenomena: **the "*raisons d'être*" tend to disappear**

… and students choose to be believers or not…


Didactic co-determination
The dissemination of knowledge is affected by conditions and restrictions beyond the classroom (*scale of didactic co-determination*)
POSTULATE IN THE ATD
Individual
Collective
Can explain different didactic phenomena: didactic autism, Topaze effect, curriculum linearization, monumentalism, etc.
Institutional Subject
The role they are forced to play (the subject, such as student or teacher).

The emergence of SRP

Ecological Postulate
Study the conditions and constraints of existence and restrictions that affect the dissemination of knowledge in institutions

Didactic phenomena:
paradigm of visiting the works
(Chevallard, 2004)

INQUIRY PROCESS

The emergence of SRP

Scale of levels of co-determinacy

Humanity

↓↑

Civilisations

↓↑

Societies

↓↑

Schools

↓↑

Pedagogies

↓↑

Disciplines ↔ Domains ↔ Sectors ↔ Themes ↔ Topics

Didactic transposition:

How are contents selected and organised in topics, themes, domains, etc.

How could it be otherwise?

Ecology and economy of school praxeologies:

How do the upper levels of **the scale affect** the "life" of school praxeologies and their possible evolution?

Reference models as sequences of punctual, local or regional praxeologies

The emergence of SRP

Constraints from the upper levels

Limitations for developing the research activity:

- difficulties in describing the in vivo analysis,
- requires hard work to analyse the epistemology of the topic (including a historical point of view) to emancipate from the current epistemological model,
- new epistemological models are not sufficient to overcome curricular fragmentation (the curriculum restrictions always appear) or the constraints for the upper level of the scale of co-determinacy,
- …

Humanity
↓↑
Civilisations
↓↑
Societies
↓↑
Schools
↓↑
Pedagogies
↓↑
Didactic system $\mathbf{S}(X, Y, \mathscr{p})$

The Study and Research Pathway (SRP) — *Parcours d'Étude et de Recherche — did not emerge out of nowhere, but rather evolved from the Study and Research Activities (SRA) to address the problem of curriculum problem*.

Getting to know Study and Research Paths (SRP)

Theoretical constructs and their origin

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Getting to know SRP

First formulations

"[…] we will consider school activity, at any level, as an activity of studying questions, under the guidance of a teacher who thus assumes the role of director of study, the questions to be studied normally being included in a curriculum […]" (Chevallard, 1998, p. 7)

"[…] we denote S(X; y; Q) the didactic system […] it is possible that the study collective X, directed by Y, has to study a whole series of questions, Q, Q', Q", etc. We denote such a study program by P = {Q, Q', Q"...} and the resulting didactic system by S(X; Y; P). The "formula" S(X; Y; P) thus represents the most general case of a didactic system. " (Chevallard, 1998, p. 11)

Proposed as tools for analysing and designing:

- pure mathematics education e.g the connection between fractions and periodic decimals
- as a model for the back then new TPE (Travaux Personnels Encadrés) or as model for interdisciplinary programmes (Chevallard, 1998, 2001, 2004).

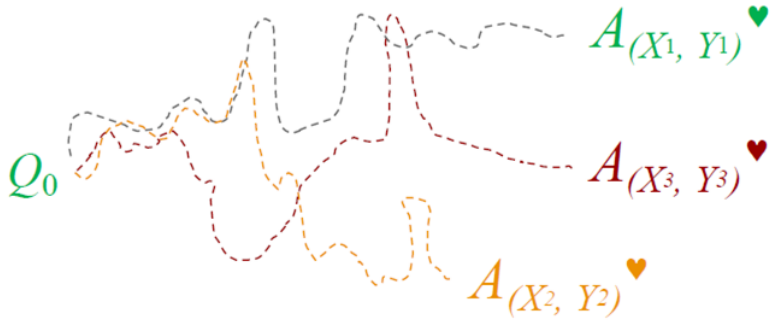
Getting to know SRP

The dynamics of the didactic system

The Herbartian schema as a didactic model of reference of SRP

- Group of **students** *X* and study **supervisor(s)** *Y*
- They start from a **question** Q_0 (the 'generating' question)
- They should elaborate their own (collective) **answer** $A^{\heartsuit}$ to Q_0
- To elaborate $A^{\heartsuit}$, the didactic system bring together the milieu *M*:

The milieu is composed by **derived questions** Q_i**,** prior learning $A^{\diamondsuit}$ (**labeled answers**), other **works** *W* and **collections of data** *D*:



As seen in where the same Q_0 (Broussea's guessing game (Brousseau, Brousseau & Warfield, 2002)) leeds to very different paths in TE in Japan and Spain (Hakamata et al., 2024)

$$[\, S(X;\, Y;\, Q_0) \rightharpoonup M = \{A_1^{\diamondsuit}, A_2^{\diamondsuit}, \ldots, A_m^{\diamondsuit}, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{n+1}, Q_{n+2}, \ldots, Q_p, D_{p+1}, D_{p+2}, \ldots, D_q\}. \,] \hookrightarrow A^{\heartsuit}$$

The inquiry process runs throughout several didactic tools

Getting to know SRP

The counter paradigm

For schools to educate well reflected citizens, student must be engaged in posing questions and pursuing answers:

"Historically, posing questions was the privilege of the mighty, although it has become a defining right of citizens; but it is a right not yet exercised as it should in a fully developed democracy. […] a citizen does not only have to be educated in many fields but, in the procognitive perspective of the new didactic paradigm, a citizen must be ready to study and learn, even from scratch, fields of knowledge new to him/her" (Chevallard, 2015, p. 181).

"… mathematical contents, just like the content of any other subject matter, should not be taught as if their value and importance were taken for granted. On the contrary, they need to be constructed and appear for the students as true answers to real questions." (Barquero & Bosch, 2015, p. 260)

Getting to know SRP

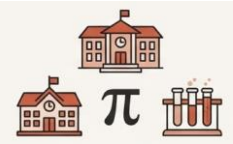
Moment of technological-theoretical construction

"**A skholè**, if I may say so, is therefore **organised around the study of a number of questions Q** to which the skholè's students seek to give answers A. Two aspects have to be emphasised here. First, for that **"scholastic" process of study not to be imposed on the students**, it is necessary for the questions Q to be regarded – by the students, by their teachers, and, so to speak, by the skhole's "board of trustees" – **as crucial to a better understanding and mastery of their lived world**. Second, in studying questions Q, students will **have to investigate many other, derived questions Q',** dynamically raised by the study of Q. Addressing these derived questions will **lead to the transposition of many praxeologies A',** that will answer many unintended questions. In the long run, this basic phenomenon **will turn the students (and their teachers) into "scholars" of a sort**." (Chevallard, 2006, p. 27-28)

Loosely speaking, learning occurs when the relation, R, one person, x, has to an object, O

$$R_i(O, x)$$

Changes under the given conditions, $\mathcal{C}$, and due to a set of gestures, Δ, enacted under these conditions (Chevallard, 2024).



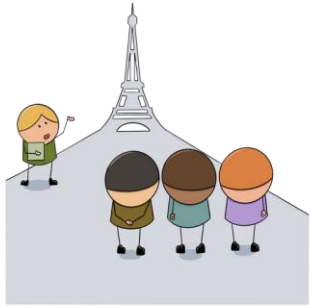
Principle for designing for SRP

The emergence of SRP

The counter paradigm

Paradigm of visiting Monuments (PVM)

Humanity
⇃↾
Civilisations
⇃↾
Societies
⇃↾
Schools
⇃↾
Pedagogies
⇃↾



Require us to emancipate from disciplinary isolation

Paradigm of Questioning the world (PQW)

Humanity
⇃↾
Civilisations
⇃↾
Societies
⇃↾
Schools
⇃↾
Pedagogies
⇃↾

Disciplines ↔ Domains ↔ Sectors ↔ …
⇃↾
Didactic system $S(X, Y, \mathscr{p})$

Didactic system $S(X, Y, Q_0)$
⇃↾
Disciplines ↔ Domains ↔ Sectors ↔ …

Didactic design of Study and Research Paths (SRPs)

Methodology and examples

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Didactic design of SRP

Rough sketch of the process of implementing an SRP

Starting from a situation with a **generative question Q_0**

To validate, disseminate and develop $A^{\heartsuit}$

To formulate one's own coherent response $A^{\heartsuit}$

Propose new derived questions *Q'*, **Q''** etc. to answer Q_0

Study and **R**esearch **P**ath

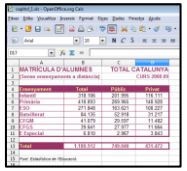
Search for answers $A_i^{\diamond}$ for ***Q'***, ***Q''*** etc. in the available ***media***

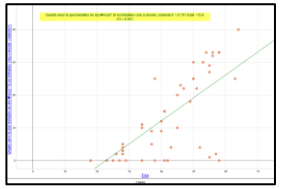
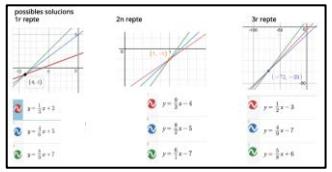
Validate (deconstruct and reconstruct) $A_i^{\diamond}$ responses using the available resources (***milieu***).

Develop the $A_i^{\diamond}$ responses and adapt them to a Q_0, ***Q'***, ***Q''*** etc.



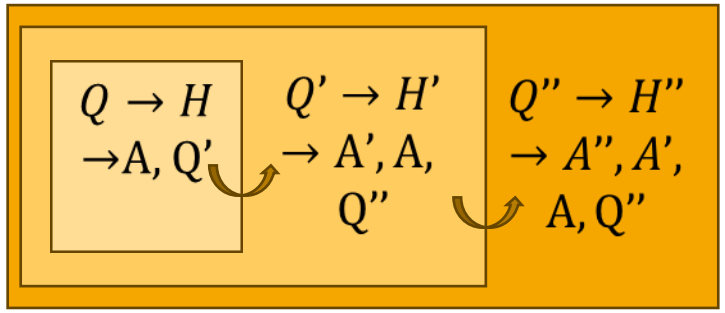


Didactic design of SRP

How to get the process moving

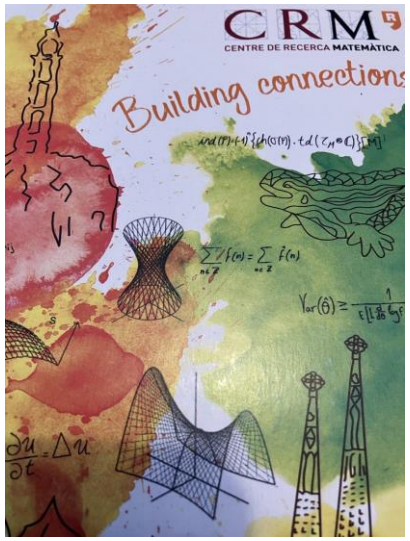


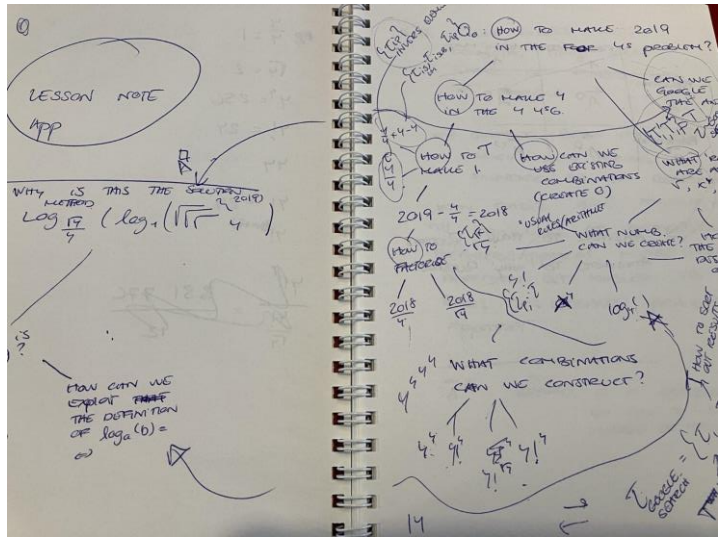
The functioning of a generating question going through (implicit) hypotheses of answers (e.g. Barquero, 2009)

How to map the path from a generating question, Q_0, to the development of a praxeological organisation, $\wp$?

Highlights 'evanescent praxeologies': intermediate pieces of knowledge that are often overlooked in traditional teaching but are essential for moving from the problem to the solution in a hard sense.

Q_0: How to make 2019 in the four 4's problem?





"how type" $\overrightarrow{Q_i}$ and "why type" $\overrightarrow{Q_j}$ adressing the Π and the Λ of the $\wp$ (Otaki, 2022).

As a design principle: How to solve… ; why is … the solution.

Didactic design of SRP

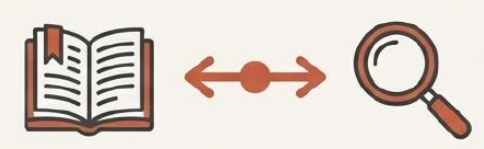
The role of transmission of knowledge

Regarding educational systems:

"Human societies cannot afford to discard the knowledge accumulated by previous generations […] students should not only learn to attack problems in a "bare-handed way"; they should also learn to search for knowledge in books and other resources, and make use of this knowledge." (Winsløw, Matheron & Mercier, 2013, p. 267).

Didactic design of SRP

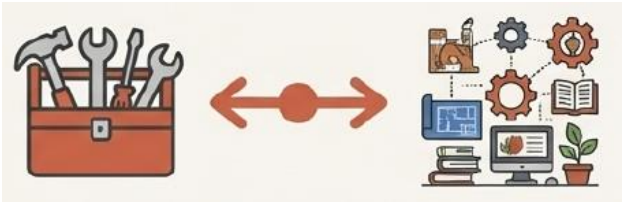
The role of transmission of knowledge

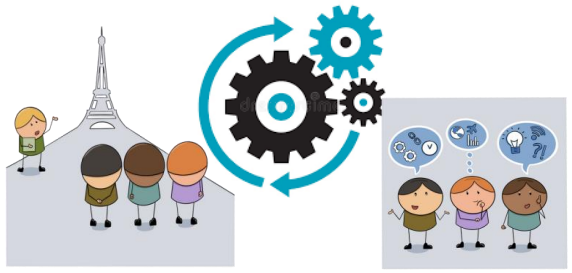


In essence, the process of inquiry "…inquiry, in "my" sense, comprises necessarily **episodes labelled "direct" as well as episodes labelled "inquiry-based."** Both of them are part and parcel of the same process, that of inquiring. […] "When inquiring about question *Q*, one will surely inquire about existing answers: this is what scientists would do, as well as any reasonable layman outside the classroom." (Chevallard, 2008, p. 3).

Answers which can be validated against the milieu.

"**The didactic milieu of the Herbartian formula can include an a-didactic milieu** […], that is, a system of objects acting as a fragment of "nature" for *Q*, able to produce objective feedback about its possible answers without any didactic intention towards *X*." (Kidron et al., 2014, p. 157)

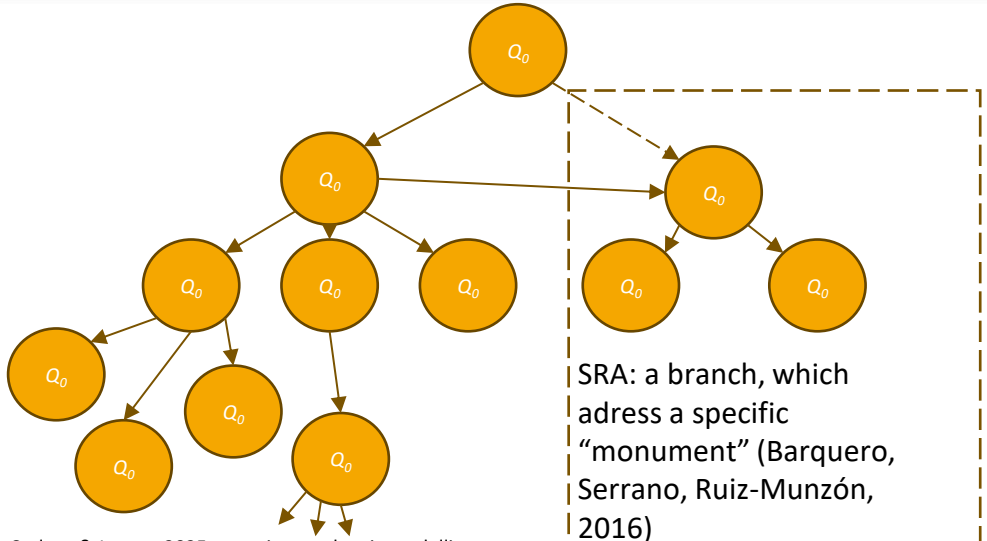




The goals of education are not just to acquire certain modes of work and thought. In reality, specific "pieces" of knowledge are included in what the student should acquire (Winsløw, Matheron & Mercier, 2013, p. 267).

Didactic design of SRP

The mapping of the process



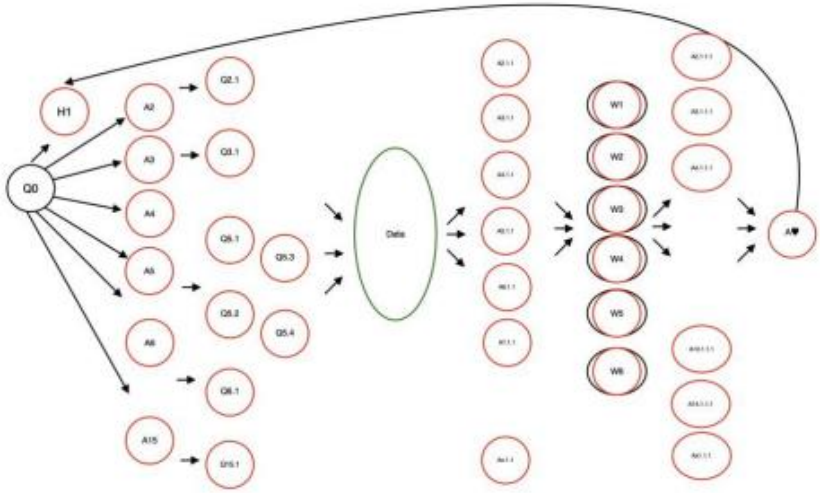
Carlsen & Jessen, 2025, mapping stochastic modelling

SRA: a branch, which adress a specific "monument" (Barquero, Serrano, Ruiz-Munzón, 2016)

Q_0 "Which exam format is best?"

Objective 1: students to study research designs to plan statistical analyses, and how such a type of inquiry has been modelled before.

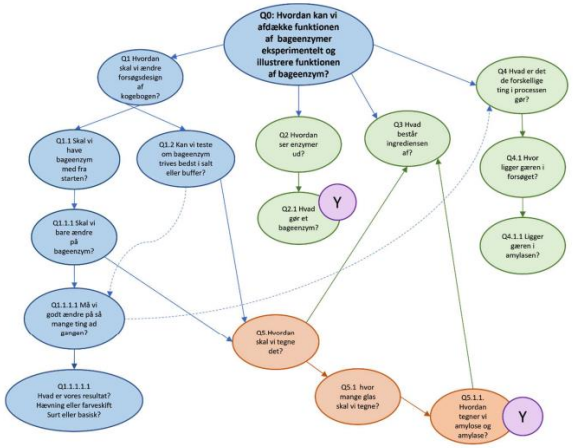
Objective 2: based on D_1, answer Q': "What can this data set tell us about which exam format is best?" (Carlsen & Jessen, 2025)



Design feature

Open inquiry leads to organisation of data collection.

Inquiring statistical descriptors, teacher guided "why questions". (Østergaard & Larsen, 2023)



Modelling enzyme kinetics

Blue: design of experiment

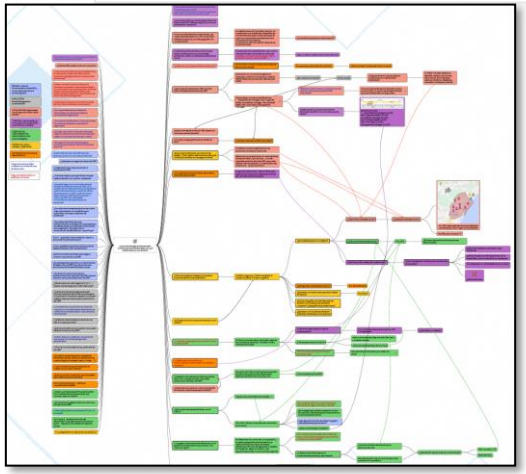
Green: theoretical knowledge

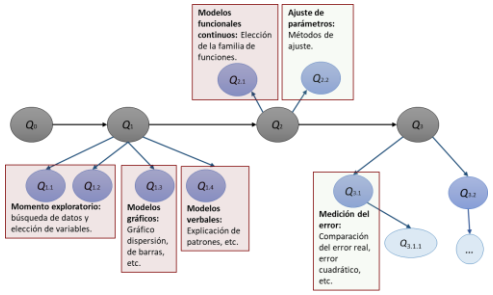
Orange: Lab work as simulation

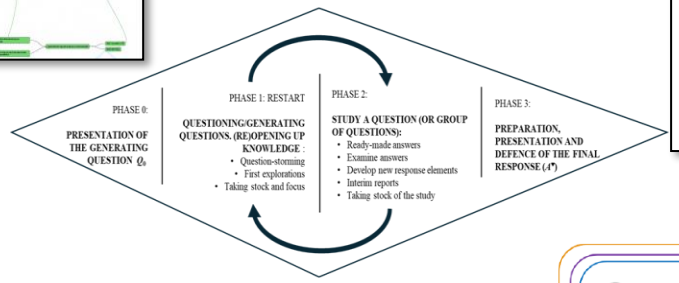
Purple: teacher prompts (Jensen & Puge, 2024)

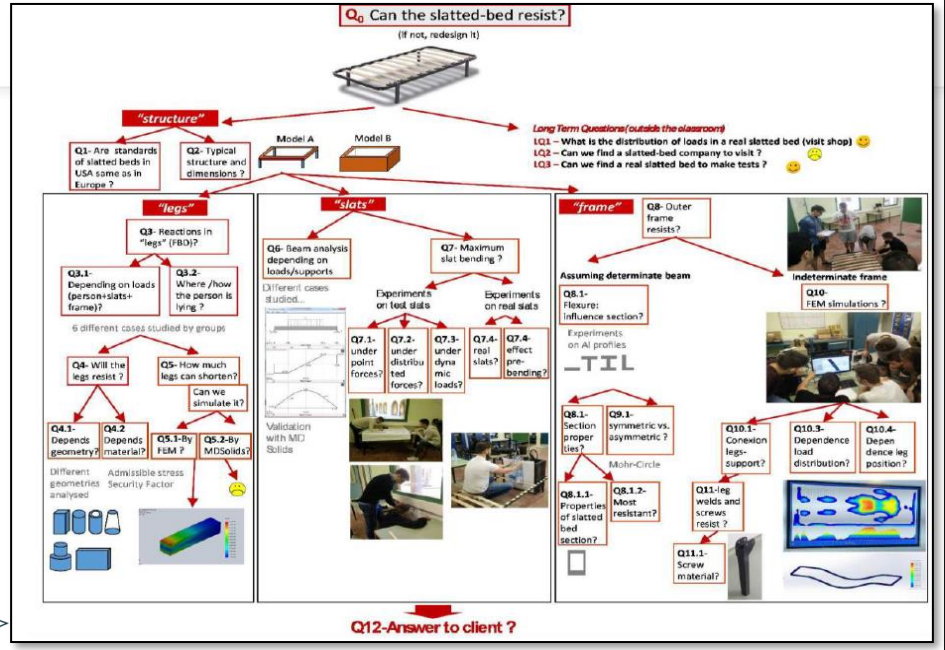
Getting to know SRP

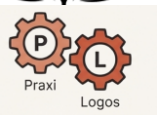
Moment of exploration

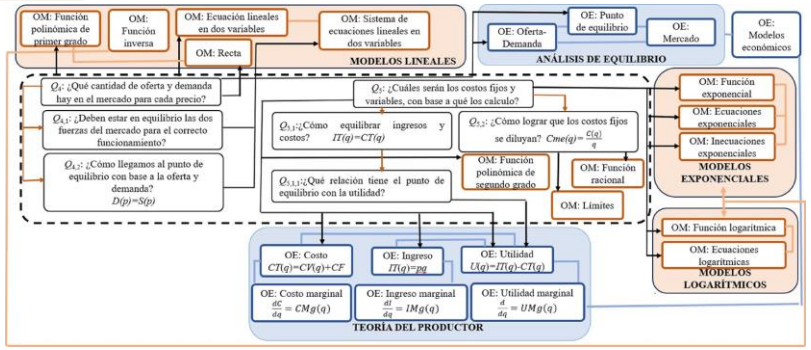


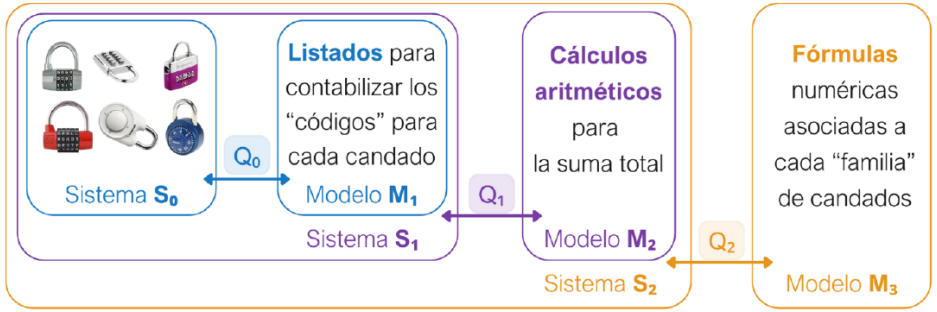




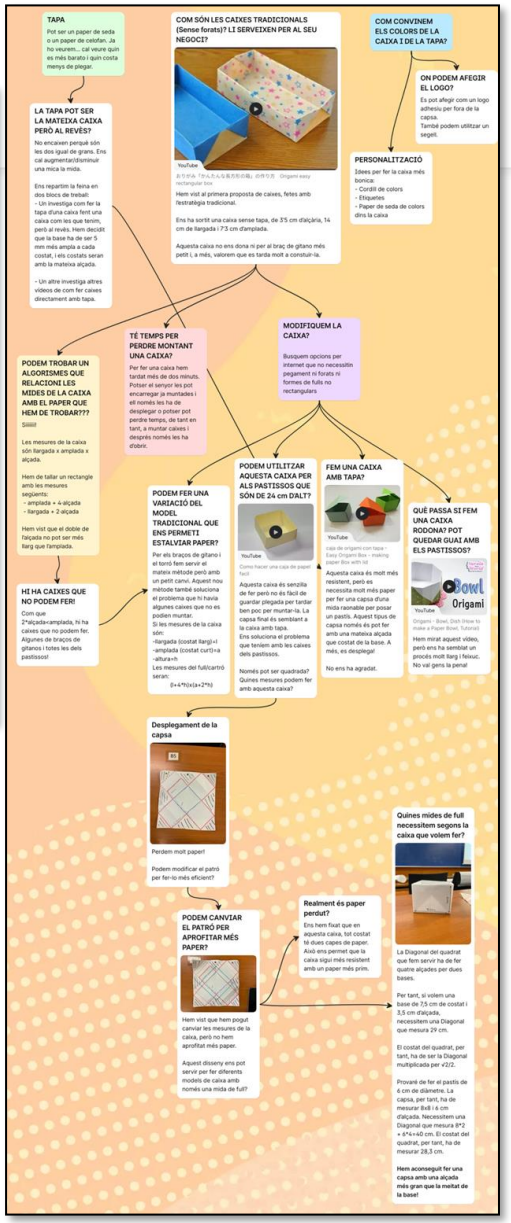








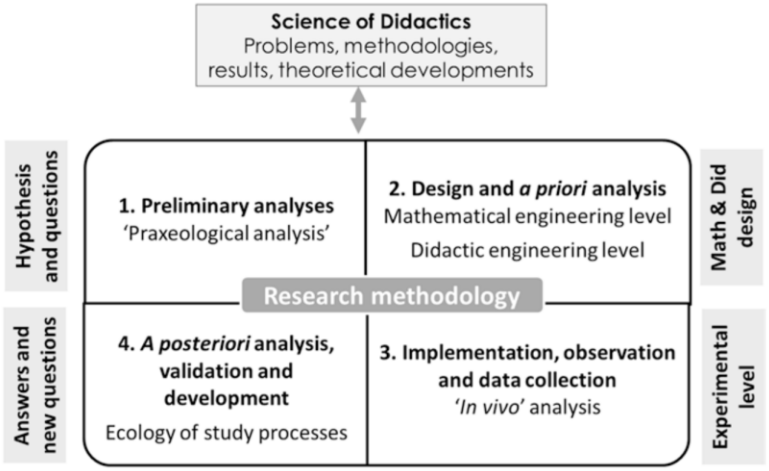
Q-A maps



Didactic design of SRP

Rough sketch of the process of implementing an SRP

Research practises on SRP can guide didactics research how to analyse and study "[…] the possible raisons d'être of mathematical contents within the students' reach […]." (Barquero & Bosch, 2015, p. 260)



How to develop knowledge in an SRP?

- Whether students develop mute, weak and sound praxeologies? (Barquero, Bosch, Wozniak, 2024)
- Does it depend on the different modalities to integrate in the PVM?
- How to identify the keys to their evolution? Are there differences between "finishing" SRP and "open" SRP? Which new didactic and pedagogical contracts are necessary to develop?

Study and Research dialectics
(chronogenesis, mesogenesis and topogenesis)

Didactic design of SRP

Rough sketch of the process of implementing an SRP

"Adhering to the notion of dialectics as a "theory of contradictions," and considering the metaphorical nature of science in general and didactics in particular, we assume that the actions or practices that occur in the development of a class can be framed within different dialectical gestures: those of study and research; of the individual and the collective; of praxeological analysis-synthesis and didactic analysis-synthesis; of topic and off-topic; of the parachutist and the truffles; of black boxes and clear boxes; of media and media; of reading and writing; and of dissemination and reception (Chevallard, 2001, 2013b). It is important to emphasize that there is not duality in a dialectic, but rather an interactive process, an interrelation between its poles, which generates "something new"."

(Parra & Otero, 2018, p. 6)

Dialectics of Study and Research Paths (SRPs)

And how they are researched

euss School of Engineering
TecnoCampus
upf. Universitat Pompeu Fabra Barcelona
UNIVERSITY OF COPENHAGEN

Getting to know SRP

Moment of exploration

Early Childhood education | Primary School | Secondary School | University | Others

How should we feed our silkworms mulberry leaves to ensure they grow and develop properly?

Raisons d'être of mathematical for **Arithmetic, Cardination, Measure**

Analysis of praxis and didactic logos

(D7) Dialectics of media and milieu

(D9) Dialectics of dissemination and reception

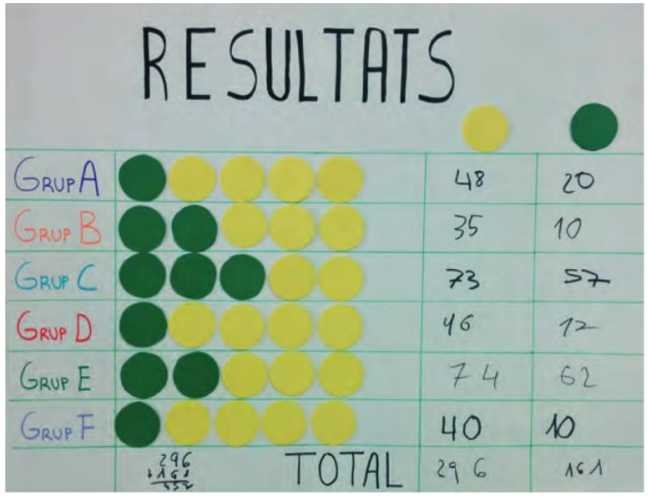
(D10) Dialectics of systems and models

$$S_0 \searrow \mathcal{W}_1 \nearrow S_1$$

Getting to know SRP

Moment of technical-work- moment of institutionalisation

Early Childhood education | **Primary School** | Secondary School | University | Others

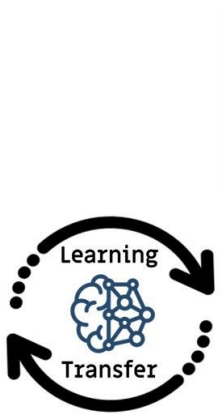


What is hidden inside the bottle?

Are we physically active?

Raisons d'être of mathematical for **Inferential statistics**

Analysis of praxis and didactic logos



(D2) Individual and collective dialectics

(D3) Dialectics of praxeological analysis-synthesis

(D7) Dialectics of media and milieu

(D9) Dialectics of dissemination and reception

Getting to know SRP

Moment of technological–theoretical- Moment of institutionalisation

Early Childhood education | Primary School | Secondary School | University | Others

Which padlock is the most secure?

Why did the Movediza Stone in Tandil fall down?

What conditions make a graph bicursal?

Raisons d'être for combinatory

Which mobile phone tariff is best for your profile?

What savings plan is best for financing an end-of-year trip?

How can we determine the break-even point, and by what amount will it vary?

Raisons d'être for calculus

How can we design and build a container with a capacity of one litre?

Raisons d'être for geometry

How can a dog breed be predicted from an image?

How does Shazam recognise music?

How can we relieve pain with paracetamol?

How much of all the information that has been said about COVID-19 has turned out to be true?

...

The generating power of Q

Analysis of conditions and constraints for SRP

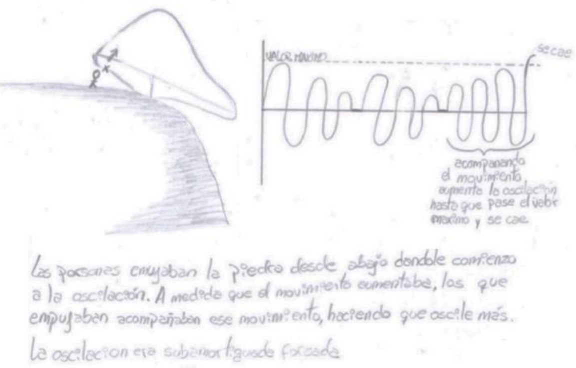
$$[\, S(\text{  }, Y, \text{  }) \hookrightarrow M = \{A_1^{\Diamond}, A_2^{\Diamond}, \ldots, A_m^{\Diamond}, W_{m+1}, W_{m+2}, \ldots, W_n, \,] \hookrightarrow A^{\blacktriangledown}$$

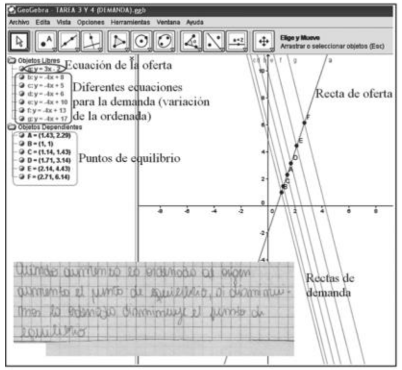
$$Q_{n+1}, Q_{n+2}, \ldots, Q_p, D_{p+1}, D_{p+2}, \ldots, D_q\}.$$

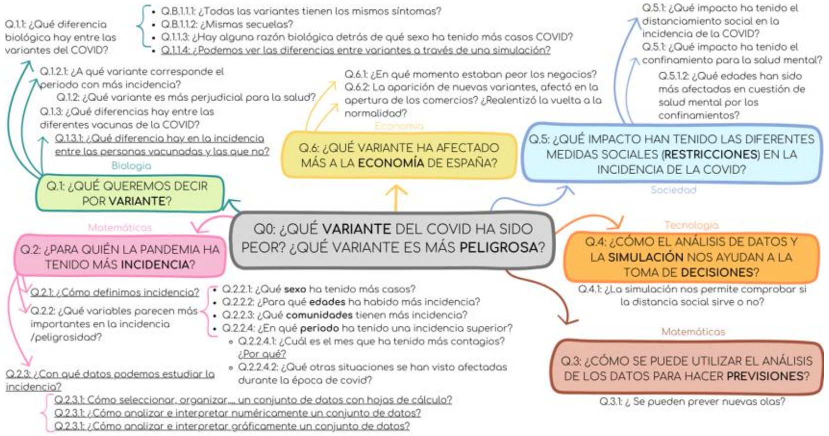
Getting to know SRP

Moment of technological–theoretical- Moment of institutionalisation

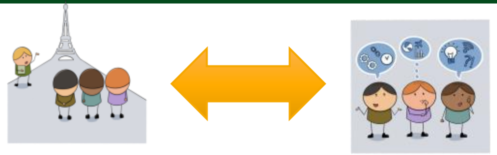
Early Childhood education | Primary School | **Secondary School** | University | Others







Construct a new didactical contract



New tools for teaching: QA map, collaborative tools, mid-term and report, class diary, notes of teacher-researcher or external observation,...

[$S($  $;\ Y,$  $)$ $\curvearrowright$ $M = \{A_1^\diamond, A_2^\diamond, \ldots, A_m^\diamond, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{n+1}, Q_{n+2}, \ldots, Q_p, D_{p+1}, D_{p+2}, \ldots, D_q\}$] $\hookrightarrow A^\heartsuit$

Getting to know SRP

Moment of evaluation

Early Childhood education | Primary School | Secondary School | University | Others

How can we design an aqueduct to obtain the highest possible flow from a rectangular metal sheet?

Which variant of COVID-19 has been the worst?

How can we predict population growth?

How can Desigual's sales be predicted?

How can indirect taxes be managed in an audit?

How can we efficiently manufacture a slatted bed base?

How can we model and adjust real data on Facebook users and make predictions about their evolution?

Are boys better than girls in mathematics?

Why do babies die of heat stroke in cars parked in the sun?

Raisons d'être for several disciplines (chemistry, biology, physics, accounting, finances, mechanical engineering, statistics, ...)

Is a derivative function continuous?

How to measure water quality?

What is the impact of ZBE in BCN?

Analysis of conditions and constraints for SRP

Klein's discontinuities

New reference epistemological models

Teachers are experts in their discipline, but not in didactic research.

Getting to know SRP

Moment of evaluation

Early Childhood education | Primary School | Secondary School | University | Others

How can we teach the purpose and usefulness of number systems?

Health - risk or chance?

Pre-service teacher

How can we introduce the purpose of negative numbers?

How can we teach mathematical modelling in primary school?

How can we teach conics?

How can we teach our subject from a modelling perspective?

How can we introduce the purpose of real numbers?

How can we introduce probability and inferential statistics?

How can we teach proportionality?

How can we teach mathematical modelling?

Analysis of conditions and constraints for SRP- TT

Praxeological equipment for teachers

Transposition of didactic knowledge

Getting to know SRP

Moment of exploration

Early Childhood education | Primary School | Secondary School | University | **Others**

Didactic system $S(X, Y, Q_0)$

X = students with cognitive disabilities

Analysis of assumptions about responsibilities

Humanity
⇵
Civilizations
⇵
Societies
⇵
Schools
⇵
Pedagogies
⇵

MOOC → the relation between of X and Y are asynchronous

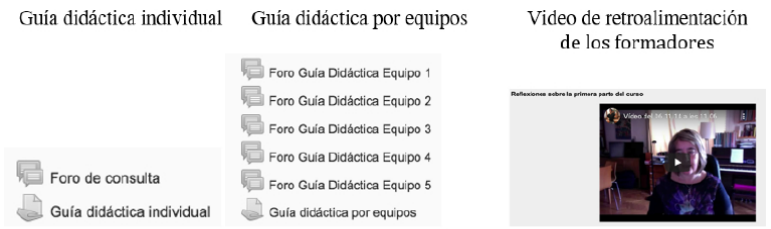


Figura 6. Actividad 2 en la plataforma

Romo-Vázquez, A., Barquero i Farràs, B., & Bosch, M. (2019).

EMP Educação Matemática Pesquisa
ISSN 1983-3156
A2
http://dx.doi.org/10.23925/1983-3156.2020v22i4p301-308

An online course to teach ATD research praxeologies: The ICMI Awardees Multimedia Online Resources

Analysis of assumptions about didactical moments

Adapt new didactical techniques of “collaborative learning”

SRP as an object of study

Moment of exploration

Problem of ecology and dissemination of SRP

- What conditions and restrictions are required for an SRP to be sustainable over time?
- Do SRPs manage to overcome thematic autism between subjects and disciplines?
    - Any transformative power?
- Do SRP overcome the phenomenon of applicationism?
- What is the impact of the “real” or “fake” question?
- What is required for the dissemination of SRPs? New infrastructure? New didactic abilities?
- Which praxeological equipment is necessary to manage the SRP?

→ **Study and research path for teacher training (SPR-TT)**

Study and Research Paths (SRPs) as an object of study

Methodology and examples

School of Engineering
UNIVERSITY OF COPENHAGEN

SRP as an object of study

Moment of exploration

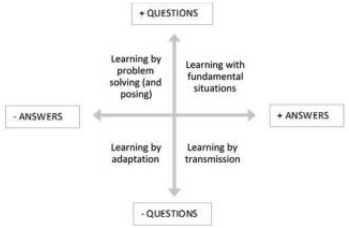
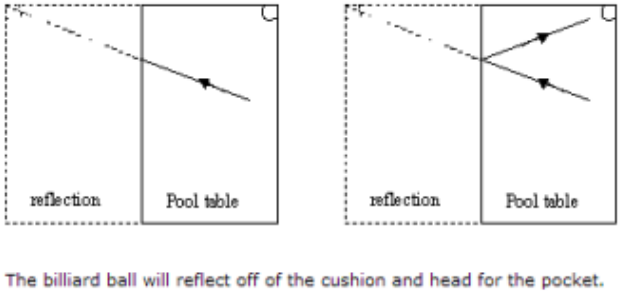


Figure 1. Four pedagogical paradigms defined by the role given to questions and answers

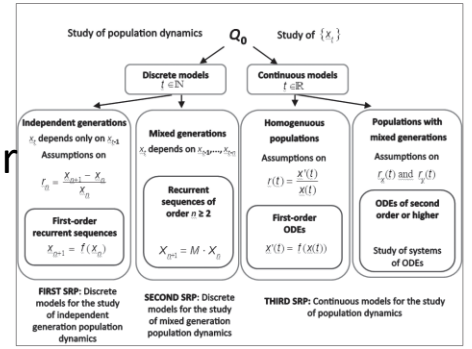


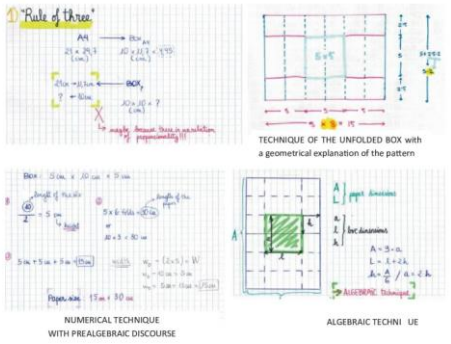
The billiard ball will reflect off of the cushion and head for the pocket.

When it comes to the practical realization of sustainable research and study processes, some important questions remain open:

1. What are the didactic and mathematical infrastructures (and resources), as well as the associated knowledge, required for the design, monitoring and evaluation of sustainable study and research processes?
2. What are the institutional conditions needed for teachers to design and implement sustainable study and research processes, and for students to engage in them?
3. What kinds of constraints or even obstacles do institutions and societies commonly offer to such processes? (Bosch & Winsløw, 2016, p. 33)

How to overcome applicationism, thematic autism between subjects or disciplines?
(Barquero, Bosch & Gascón, 2013)



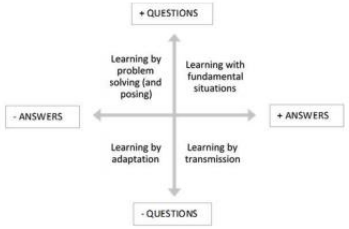


Models, techniques as named by the teacher–students (translated by the authors)

How can we go beyond mute praxeologies? How can we support further development of weak praxeologies?
(Barquero, Bosch & Wozniak, 2024)

SRP as an object of study

Moment of exploration



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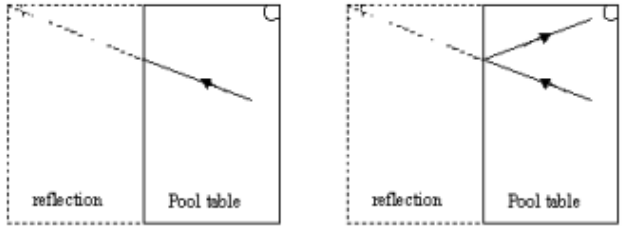

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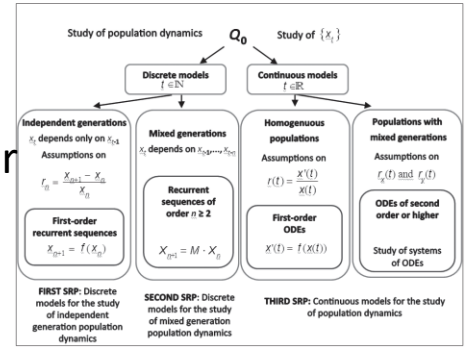

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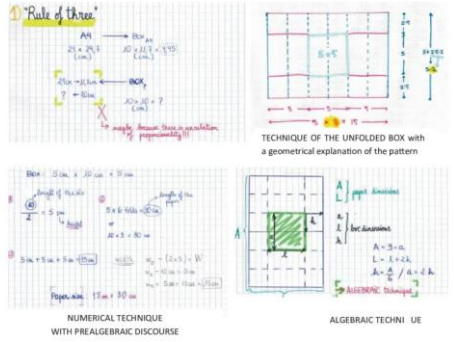
2. What are the institutional conditions needed for teachers to design and implement sustainable study and research processes, and for



Models, techniques as named by the teacher–students (translated by the authors)

How can we go beyond mute praxeologies? How can we support further development of weak praxeologies?
(Barquero, Bosch & Wozniak, 2024)

"As we have illustrated in this chapter, the conditions and constraints of SRPs originate from different institutions and constitute a very complex system to navigate."
"[…] some common constraints: entrance examinations and teachers' scholarship, which are optimised for the paradigm of visiting works." (Jessen, Otaki, Miyakawa, Hamanaka, Mizoguchi, Shinno & Winsløw, 2016, p. 135)

SRP as an object of study

Moment of exploration

In teacher education SRP has been experimented with

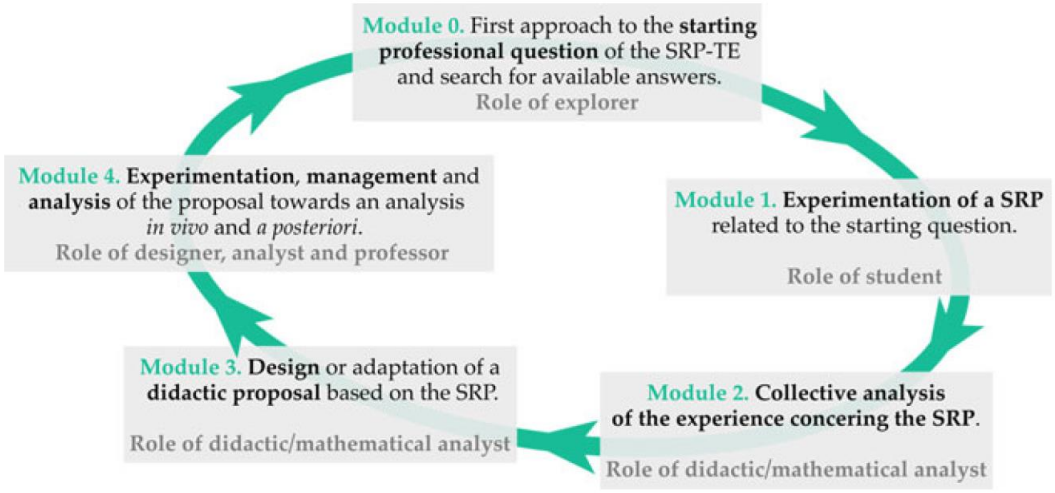
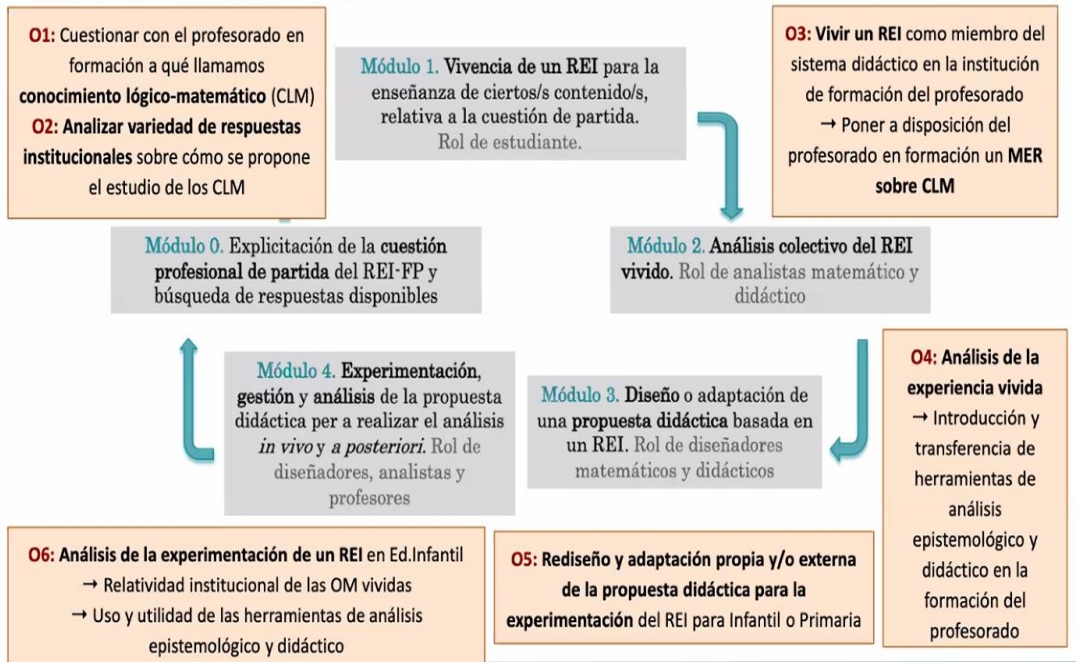


Fig. 1 Modular structure of a SRP-TE (Ruiz-Olarría, 2015; Barquero et al., 2022)

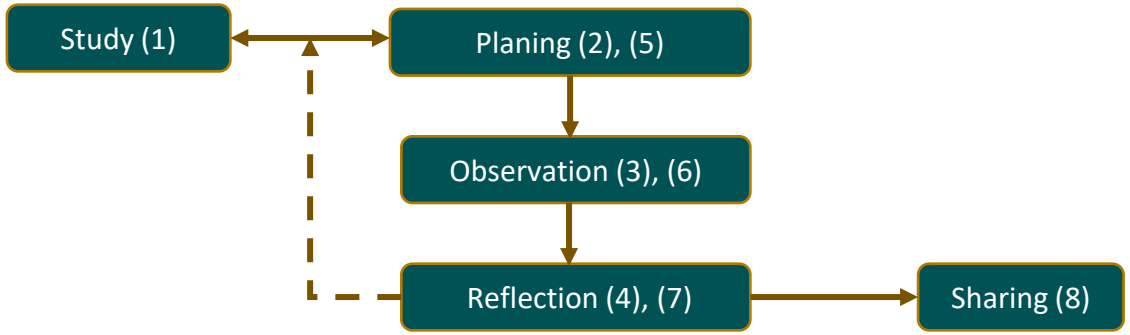


However, as studies show it is not always easy to implement in classrooms, shared trait with IBME (e.g. Primas, 2013; Rasmussen, 2016; Barquero, Bosch & Romo, 2015).

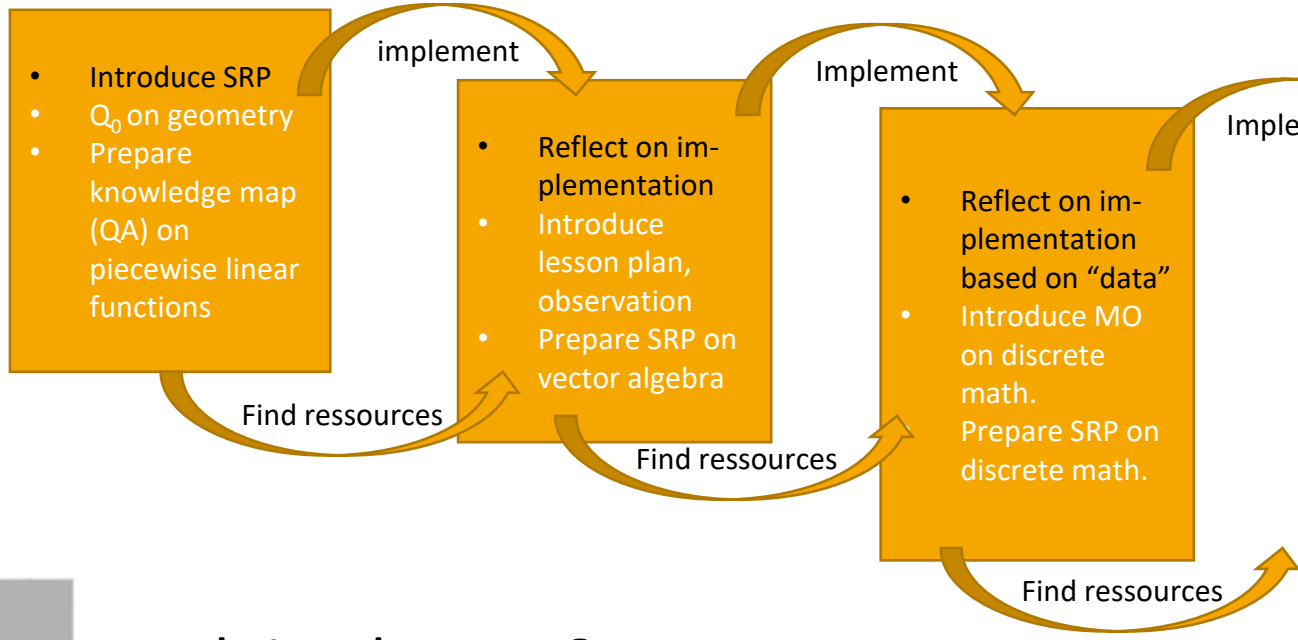
SRP as an object of study

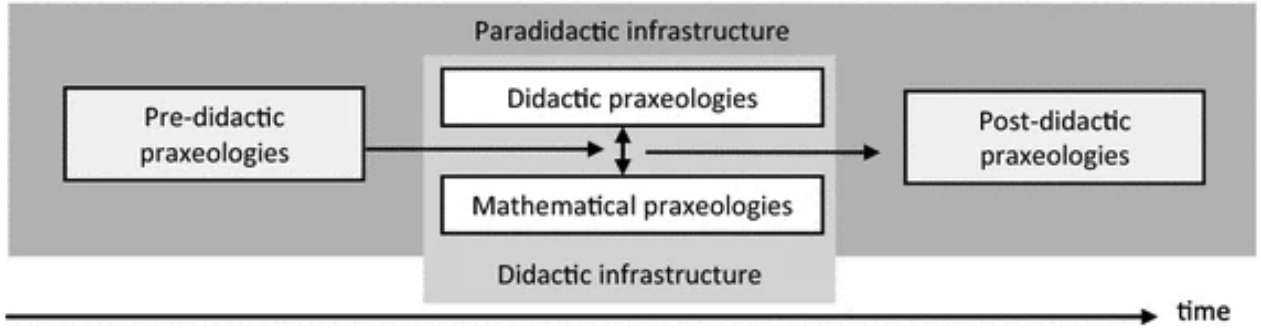
Limitations and how to move beyond

The elements of Lesson Study process



Stigler & Hiebert (1999), Isoda, Stephens, Ohara & Miyakawa (2007, p 40), Bašić (2020, p. 13)





Rasmussen, 2016, p. 307, after Miyakawa & Winsløw, 2013)

Math in Change & MatPro

81 teachers, upper sec. math.

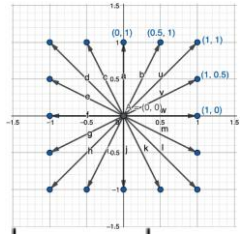
40h, 74 implemented "SRPs"

(Jessen, 2019, 2026, Jessen et al., 2016; Jessen & Rasmussen, 2020)

SRP as an object of study

Limitations and how to move beyond

Didactic tools and tools for research: Digital technologies offers a rich milieu for students to explore, experiment and test or validate evanescent praxeologies - half, preliminary or else mute answers



Q_0: …What orders would you give your crew and when do you expect to arrive? You are expected to use the notion of vector in you answer

Created an optimal route

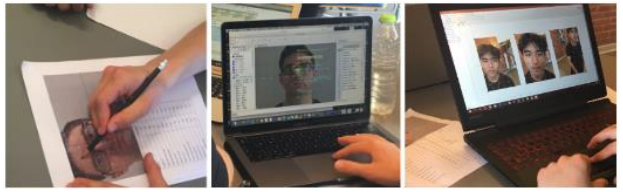
$$c_1 \cdot \overrightarrow{u_1} + c_2 \cdot \overrightarrow{u_2} + c_3 \cdot \overrightarrow{u_3} \ldots$$

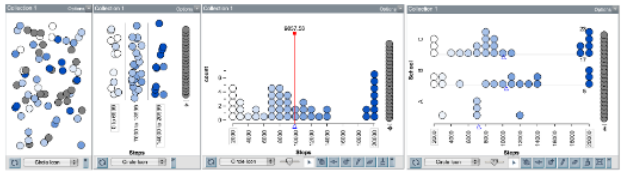
T: "Didn't you encounter the notion of 'unit vector' when reading parts of the suggested pages in the textbook? Did you notice what defines a unit vector?"

Black box uses of notion of vector, later clear boxes when combining GeoGebra syntax with textbook google searches, realising:

$$|c_1 \cdot \overrightarrow{v_1} + c_2 \cdot \overrightarrow{v_2} + c_3 \cdot \overrightarrow{v_3} \ldots| \neq c_1 \cdot |\overrightarrow{v_1}| + c_2 \cdot |\overrightarrow{v_2}| + c_3 \cdot |\overrightarrow{v_3}| + \cdots$$

(Jessen & Kjeldsen, 2023)





Prior knowledge, $\boldsymbol{A}^{\Diamond}$, is questioned: meaning of average, regression based on three measures provides $\mathbf{R}^2$=1, what is a measure?

(Østergaard & Jessen, 2025)

Research tool: the epistemic value of preliminary, half answers or hypotheses can be checked quick and in flexible ways using digital tools.

Design tool: how can we design for flexible a-didactic validation to nurture such behavior

SRP as an object of study

Limitations and how to move beyond

What it takes to implement SRP?

	SRP (Vásquez et al., 2021)	SPSL (Asami-Johansson, 2021)
X	Two groups of 30 students of grade 10	41 students of grade 7
Y	The first author of the selected paper acting as mathematics teacher and researcher, with two more mathematics teachers.	One experienced mathematics teacher
Q	Which padlock (among 6-7 different types) is safer?	Which of the surface area of the cones A (with 6 cm generatrix and 4 cm diameter of its bottom circle) or B (with 4 cm generatrix and 6 cm diameter of its bottom circle) is the largest?
Initial *M*	Students are given 6-7 different types of padlocks, each utilizing distinct code mechanisms, such as numerical combinations, letter sequences, directional inputs, and more. Students had not been introduced to combinatorics.	The teacher displayed figures of the two cones and their sizes on the blackboard. At the students' request, the teacher picked up a 3D cone and used a projector to demonstrate what it would look like when rotated. He then cut out the cone figures and showed what they looked like in a plane view, i.e. the net drawing.

Table 1. Didactic systems in the selected SRP and SPSL

Using SRP, Herbartian schema to compare and contrast SRP activites with those of Japanese Structured Problem Solving Lessons (SPSL) to discuss the constraints and conditions seen from a Japanse perspective.

"In particular, the expression "didactic transposition" indicates the transposition originating from the institutional transfer of knowledge from a scholarly institution to a didactic institution, i.e. a school system. But this is not everything. There is another form of "didactic" transposition, which has been called archedidactic transposition of knowledge (Artaud & Bourgade, 2022; Strømskag & Chevallard, 2024). An archedidactic transposition of knowledge signifies any institutional transposition from a producing institution to an applying institution […] In our case, the transposition under study involves the archedidactic transposition of the theory of SRPs from the ATD research community to the Japanese teaching profession. As we have seen, this transposition has been results of the study of confined SRAs rather than open SRPs in the teaching profession, that is, the transposition from the scholarly knowledge of SRPs to the teacherly knowledge of SRAs."

Otaki, Barquero, Asami-Johansson & Mizoguchi, 2025, p. 166

SRP as an object of study

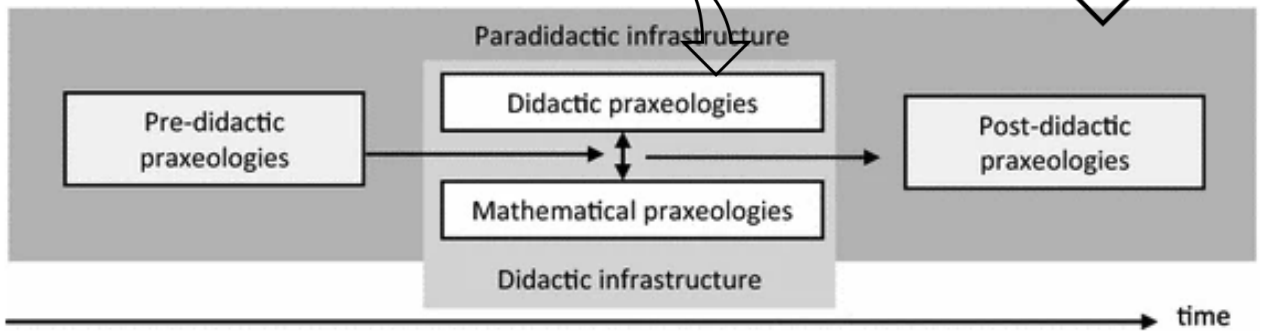
Limitations and how to move beyond

What it takes to facilitate teachers' learning processes?

$$S_{ATD}(X; Y; Q_0) \lceil^{\curvearrowright} M = \{A_i^{\circ} \dots, W_j \dots, Q_n, \dots, D_m, \dots\} \hookrightarrow A_{researchers}$$ ❤️

$$S_{PD}(X; Y; Q_0) \lceil^{\curvearrowright} M = \{A_i^{\circ} \dots, W_j \dots, Q_n, \dots, D_m, \dots\} \hookrightarrow A_{teachers}$$ ❤️

$$S_{classroom}(X; Y; Q_0) \lceil^{\curvearrowright} M = \{A_i^{\circ} \dots, W_j \dots, Q_n, \dots, D_m, \dots\} \hookrightarrow A_{students}$$ ❤️



Rasmussen, 2016, p. 307, after Miyakawa & Winsløw, 2013)

RQ_0: What kind of facilitation and questioning is needed during PD for teachers to realise the learning potentials of SRP?

$Q_{0,PD}$: How can we introduce vector algebra based on a Q_0, where students acknowledge the need for studying W_j?

$Q_{0,CR}$What orders would you give your crew and when do you expect to arrive? You are expected to use the notion of vector in you answer.

SRP as an object of study

Limitations and how to move beyond

Facilitating teachers' implementation of SPSL (Bahn, Asami-Johansson & Jessen, 2025)

Researcher considered the following questions during reflection meeting (identify if there is an overlap between teachers' observations and F_{ext}'s learning goal for the teachers):
Q_1: How many students did actually realise the connection between angle and length by themselves?
And the mathematical question of $Q_{1,1}$: What is the connection?

Expert teacher, who facilitated the reflection meeting:
Q_1: Can we do something to get students' conversations going?
Q_2: Could we do something to make the goals clearer to the students? Q_3: Something else?

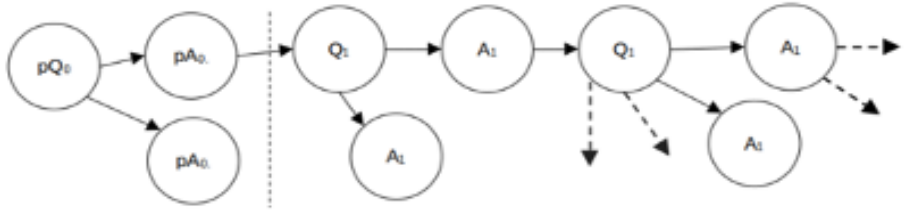
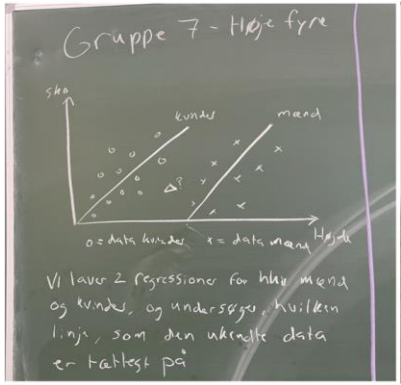


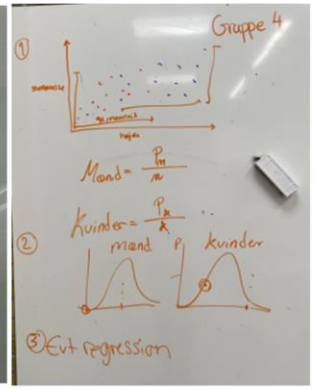
Figure 2: Illustration of the QA-map of F_{ext}'s a priori analysis. The dotted line indicates a shift from paradidactic QAs to didactic QAs

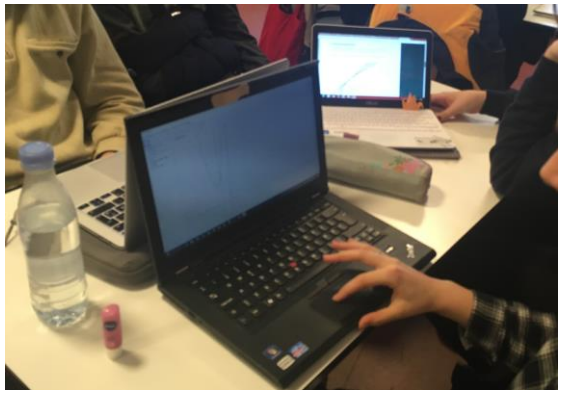
SRP as an object of study

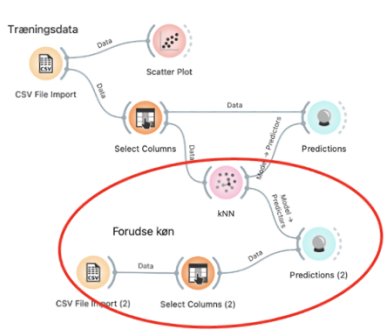
Limitations and how to move beyond

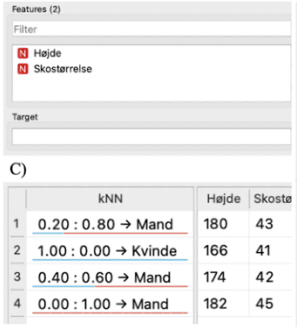
Methodologies, vocabulary, …. needed for in vivo analyses, orchestration…



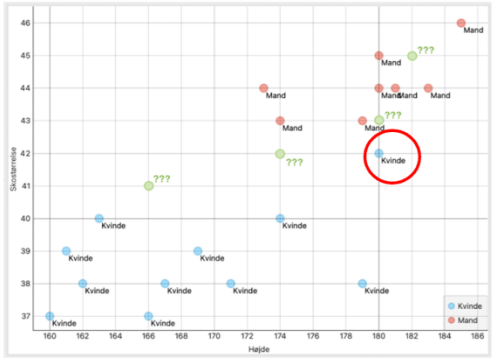








	kNN	Højde	Skoste
1	0.20 : 0.80 → Mand	180	43
2	1.00 : 0.00 → Kvinde	166	41
3	0.40 : 0.60 → Mand	174	42
4	0.00 : 1.00 → Mand	182	45



(Tarp, Frantzen & Jessen, 2025)

Zoom meetings record both screen actions and sound from students.

➡️ map the QA-dialectics, the study-research dialectic, the individual-collective dialectic.

We still need research which shows the connections between

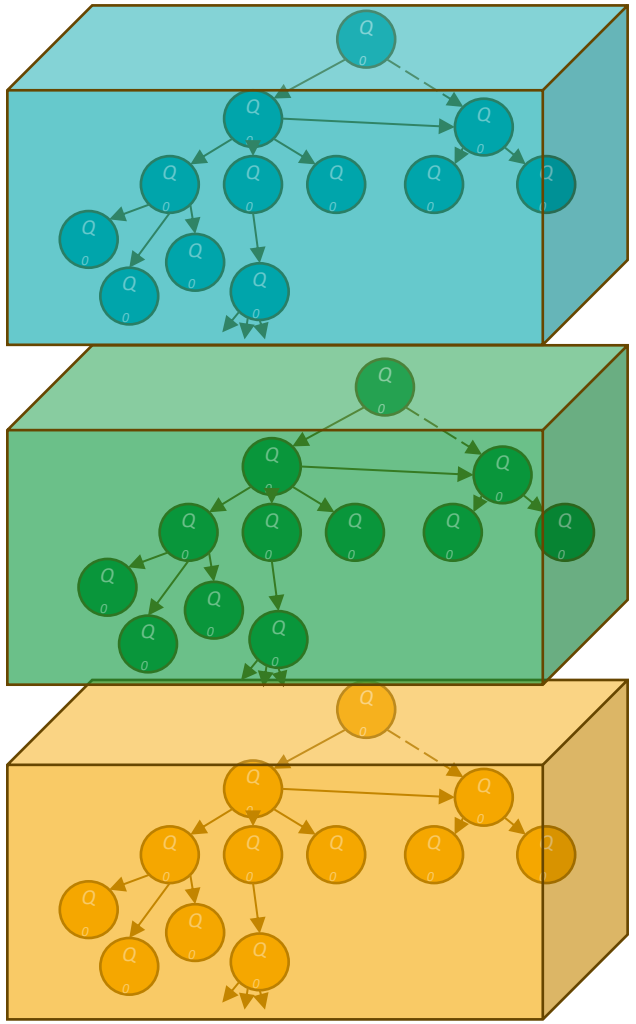
$$S_{PD}(X; Y; Q_0) \leftrightsquigarrow S_{classroom}(X; Y; Q_0)$$

Do we see the growth of practice knowledge within teacher communities as anticipated?

What can it tell research about design principles for nurturing autonomous questioning among students?

SRP as an object of study

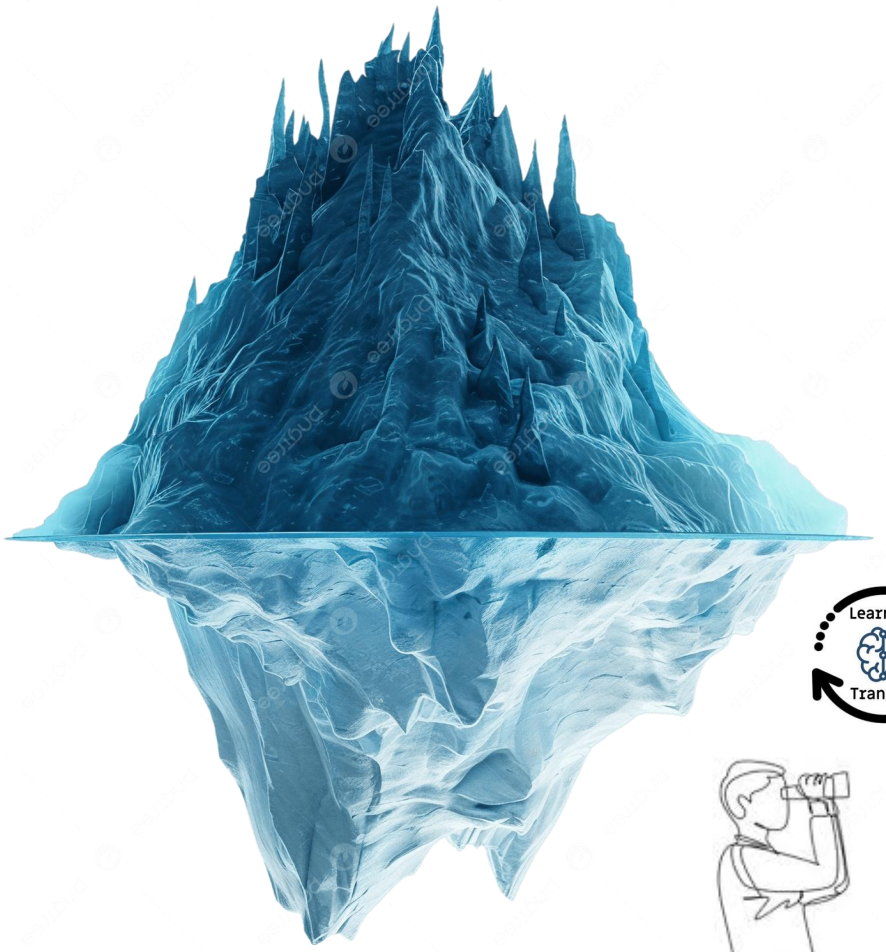
Moment of exploration



Researchers' learning process

Teachers' learning process

Students' learning process



SRP as an object of study

Moment of institutionalisation

Problem related to the infrastructure for the SRP research

EducMath

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AMPERES

— Dernière modification 24/11/2009 08:22

Apprentissages Mathématiques et Parcours d'Études et de Recherches pour l'Enseignement Secondaire

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Objectifs

Dynamiser l'étude des mathématiques dans l'enseignement secondaire par la mise en place d'Activités ou de Parcours d'Étude et de Recherche. Développer des activités et des parcours d'étude et de recherche qui redonnent du sens aux mathématiques enseignées dans le second degré. Etudier les conditions de réalisation effective de telles activités et de tels parcours.

http://educmath.ens-lyon.fr/Educmath/recherche/archives/equipes_associees/didactique

Labinquiry

https://www.ub.edu/labinquiry/es/


Thank you for your attention