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Conditions and Constraints of Professionalisation: Advances from the Anthropological Theory of the Didactic

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Outline

I. Introduction: Professions and main problematics

II. Long story since the 1st CITAD: our unit of analysis

III. *Herbartian* research questions and *research praxeologies*

IV. Concluding remarks and future developments

I. Introduction: Professions and main problematics

- Axis 3 paces **professionalisation** (of teachers and beyond teachers profession) at the centre of research within the ATD.
- Some of the main concerns in Axis 3
  - Conditions and constraints of professionalisation
  - Mathematical and didactic knowledge for professional practice
  - Didactic devices for professional training
  - Professionalization in view of evolving societal affordances and new technical devices
- Not exclusively focus on **teaching profession**, covering also other professions: engineers, economists among others.
  - How can ATD contribute to the analysis and development of the transformation of professions and professional practices?

I. Introduction: Professions and main problematics

Chevallard (2011) distinguishes **three main problems**, which can be used as **foundational questions** for research

[P_1] The basic problematic → *conditions for integration*

- **Question:** Under certain constraints on a particular institution or person, under which set of conditions could that institution or person integrate into its praxeological equipment a particular praxeological entity $\wp$?

$$\{ C / \partial(K_0, C, \wp_0, U_0) \}$$

[P_2] The possibilist problematic → *praxeologies actually accessible*

- **Question:** Given a certain set of conditions and constraints to which a particular institution or person is subject, what praxeological systems are possible for that institution or person to access?

$$\{ \wp / \partial(K_0, C_0, \wp, U_0) \}$$

I. Introduction: Professions and main problematics

[P_3] The primordial problematic → *praxeological equipment*

- Question: Given a project of activity, in which a particular institution or person intends to engage, what is the **praxeological equipment** that can be considered indispensable or simply useful for that institution or person in the design and accomplishment of that project? (Chevallard, 2011, p. 98)

> [...] if recognising **praxeological needs** is a prerequisite for the evolution of teaching practices, it is only a starting point. It remains to be determined **how to respond to these needs** [and how to methodologically detect and address them]. One response, formulated by the ATD, is the establishment of teaching as a **profession**, a condition that, at the very least, is a matter for our society. (Wozniak, 2020, p.799) (our translation)

Line 1 — *Professions and their ecology*

Line 1 examines how a profession is constituted by the set of civilisational, societal, institutional conditions and constraints, and how these determine which praxeologies are recognised as legitimate components of a profession and the professional training

Several professions included:

- Teaching profession
- Research profession
- Other professions

Line 2 — *Praxeological equipment and its fragility*

Line 2 focuses on identifying and analysing the praxeological equipment required for training and for being part of a profession. It also includes the analysis of its fragility and incompleteness.

> Given a professional project Π_0, what praxeological equipment exist and what is missing or underdeveloped?

Line 3 — *Transformation of professional practices*

Line 3 focuses on the transformation of professional practices through the design, implementation and analysis of (new) didactic devices.

> Given a professional project Π_0, what conditions might be created so that the required praxeological equipment could be integrated and institutionally sustained to transform professional practices?

Line 4 — *ATD in dialogue with other theoretical frameworks*

Line 4 operates at a meta-theoretical level by analysing how these same problematics are conceptualised—or left implicit—in other frameworks, and by showing the specific contribution of the ATD.

Line 4 = Meta-analysis of {P1, P2, P3} across theoretical frameworks

II. Long story since the 1st CITAD: our unit of analysis

Following T1.1 from the plenary talk (Miyakawa & Kaspary, 2026):

- Corpus of papers [From CITAD 1 (2005) to CITAD 8 (2026)]
  - Proceedings of CITAD conference: from CITAD 1 to CITAD 8
  - Bosch, Chevallard, García & Monaghan (2020, Eds.). *Working with the ATD in Mathematics Education. A Comprehensive Casebook.* Routledge.
  - Chevallard, Barquero, Bosch, Florensa, Gascón, Nicolás, & Ruiz-Munzón (2024). *Advances in the Anthropological Theory of the Didactic*. Birkhäuser Cham.
  - Some representative papers (related to the selected cases)
- Selection of papers according to the axis defined along CITAD:

  In CITAD1: *Mathematical and didactic organisations*

  …

  to CITAD8: *ATD and the professionalization of the teaching profession*

  \+ Individual revision of the papers in other axis

II. Long story since the 1st CITAD: our unit of analysis

Selection of papers according to the axis defined along CITADs

CITAD	1	2	3	4	5	6	Book	7	IRP	8	Total
Total contrib.	36	41	41	44	36	60	13	52	27	76	426
Selected contrib.	6	12	27	15	11	17	6	17	9	15	135

II. Long story since the 1st CITAD: our unit of analysis

Each paper was described using a common analytical grid including:

- Contextual data: conference/book, year, country/ies
- Professional focus: teacher education (pre- or in-service) and school level or other professions.
- Selection of (primary and secondary) Lines of Research

  starting from the work initiated in the IRP-Topic 2 about TE
- Analysis of the Research Praxeologies, with its *praxis* and *logos* (Artigue et al., 2011; Artigue & Bosch, 2014)

$$\mathcal{R}\wp = [T / \tau] \oplus [\theta / \Theta]$$

  - Research questions (T = RQ) + Research techniques (research methods and units of analysis considered)
  - Research technological–theoretical elements

II. Long story since the 1st CITAD: our unit of analysis

Selection of lines	Elements of the research praxeology

				Research praxeologies				
Paper	Authors	Line PRIMARY	Line SECONDARY	Research questions (RQ)	Research method (R_technique)	Unit of analysis = Institutions + empirical data	Research technology-theory (Research logos)	Hypothesis | Phenomena related to
1	Pressiat, A.	Line 1: Professions. Teaching profession	Line 2: Praxeological infrastructure and equipment of teachers	What does ATD bring to the teaching profession? How are these contributions received and interpreted by different actors of the profession? What conditions and constraints shape this reception?	This is not empirical classroom research, but professional-ecological analysis	Institutions: Didactic research institutions interacting with the profession. Empirical data: Discourses on ATD about professions. Institutional positions and reactions to ATD tools. Examples of appropriation, misunderstanding, resistance.	professional problems; institutional ecology; reception of scientific knowledge	
2	Cirade, G.	Line 1: Professions. Teaching profession		What constitutes a "problem of the profession"? How do such problems emerge, persist, and interact within the ecology of the teaching profession? How can ATD concepts be used to analyse these professional problems?	Theoretical and conceptual analysis grounded in ATD, drawing on examples of professional situations to illustrate ecological mechanisms.	Institutions: The teaching profession. School and educational institutions interacting with the profession. Empirical data: Professional discourses and practices cited as examples of recurring problems. Institutional situations illustrating how problems are produced and disseminated.	professional problems; institutional ecology; didactic codetermination	Professional problems persist becau they are ecologically stabilised within institutional systems, and teacher education often reproduces rather th questions these systems.
3	Montoya González, S. & Lezama, J.	Line 1: Professions. Teaching profession	Line 2: Praxeological infrastructure and equipment of teachers	Why do professional development processes often remain local and non-reproducible? How can ATD concepts help identify the conditions under which professional development can be reproduced?	Qualitative analysis of professional development processes, interpreted through ATD concepts to identify invariant praxeological structures.	Institutions: Teaching profession in Mexico. Professional development institutions and programmes. Empirical data: Descriptions of professional development processes. Analyses of professional practices and training situations. Identification of praxeological invariants and conditions affecting reproducibility.	professional development; praxeological invariants;	Professional development becomes reproducible only when its praxeolog and institutional conditions are made explicit and shared.
4	Olivero, F., Bosch, M., & Gascón, J.	Line 2: Praxeological infrastructure and equipment of teachers		What geometric praxeologies are at stake in teacher education? How can these praxeologies be analysed and reconstructed within teacher education? What conditions affect their development and transposition?	Didactic and praxeological analysis of teacher education situations and geometric organisations, grounded in ATD.	Institutions: School and academic geometry as institutionalised knowledge. Empirical data: Teacher education situations involving geometry. Analyses of tasks, techniques, and technological–theoretical discourse in geometry. Teachers' productions and discussions in TE contexts.	REM, dominant epistemological models, mathematical praxeology (for the teaching profession); didactic transposition processes	Teacher education in geometry need explicitly work on alternative epistemological models and analyse prevailing ones.
5	García García, F. J.	Line 2: Praxeological infrastructure and equipment of teachers	Line 3: The PQW in TE	How can inquiry-based professional development modify teachers' didactic praxeologies? What conditions and constraints affect the durability of these modifications?	Design, implementation, and qualitative analysis of a professional development programme grounded in ATD, focusing on the evolution of teachers' didactic praxeologies.	Institutions: In-service teachers. Empirical data: Records of professional development sessions. Teachers' productions, discussions, and reflections during inquiry activities. Analysis of changes in didactic praxeologies over the course of the programme.	didactic and mathematical praxeology; inquiry-based learning; professional problems; didactic infrastructures	Durable modification of teachers' praxeologies occurs only when professional development is organise around inquiry into professional problems and supported by appropri infrastructures.

II. Long story since the 1st CITAD: our unit of analysis

How do we organize the presentation?

- From Line 1 to Line 4
- What are the main research topics of the line?
- What are dominant 'Herbartian' research questions in the line, i.e., research questions that seems critical for the line and that have had more research developed around.
- Selection of some representative papers linked to a Herbartian research question and related research praxeologies:

$$\mathcal{R}\wp = [T / \tau] \oplus [\theta / \Theta]$$

  - Elements of the *praxis*: specific *RQ*s and *research techniques* or methods
  - Elements of the *logos*: crucial theoretical notions from ATD, etc.
  - [Occasionally] Distinctive outcomes or cross-references

Line 1 — *Professions and their ecology*

Line 1 examines how a profession is constituted by civilisational, societal, institutional conditions and constraints, and how these determine which praxeologies are recognised as legitimate components of a profession and the professional training

Several professions included:

- Teaching profession
- Other professions:

  Engineering, Nursing, Craftsmanship, Helicopter piloting, Bricklaying

- **Research profession** (Artigue, Bosch & Gascón, 2011; Artaud, Bernad & Redondo, 2024; Fouchet-Isambard & Lombard, 2026)

Herbartian research questions in Line 1

Professions beyond the teaching profession

- $RQ_{L1.1}$: What conditions and constraints shape the emergence, delimitation, and transformation of *professions* and *professional knowledge*?

  (Ladage, 2017, 2026; Ladage, Achiam & Marandino, 2020; Castela & Romo, 2011; Paris, 2024; Mortenson, 2011; Ricardo and Pietrocola, 2011; among others)

- $RQ_{L1.2}$: How do these conditions and constraints affect the existence, legitimation and dissemination of *professional and training praxeologies* across different institutions and training contexts?

  (Giménez, Delprato, Fregona & Orús, 2019; Manceau, 2018; Rivier-Perret, 2020; Ladage, 2026)

III. Line 1: Professions and their ecology

Ladage, C. (2026). Study of the ATD as a theoretical frame of reference for research in professional training. CITAD8

Elements of ℛ praxis ⟺ *Elements of the ℛ logos*

- RQ1: How do civilisational, societal, school, … constraints shape *professional praxeologies* in vocational and professional training contexts? – Analysis of conditions and constraints constraints in didactic systems (nursery, prisons, crafts, aeronautics, digital contexts)
- RQ2: How do professional praxeologies circulate between *practice*, formal *training*, and informal learning spaces? – Study of circulation of professional praxeologies

Naturalistic techniques: observation, interview, questionnaires + inventory and questioning of reading of discourses, witness techniques for analysing existing culture.

- *Professionalisation* in the light of transformation affecting human societies.
- *Scale of didactic codeterminacy*

  Ecology / Economy
- *Cognition,* persons, and institutional positions
- *Praxeologies* and their institutionalisation in human practices

III. Line 1: Professions and their ecology

Study case 1: Knowledge useful for Web search (Ladage, 2008; Ladage & Chevallard, 2011)

RQ: What conditions and constraints do exist related to the institutional praxeological equipment in Web search?

Analysis of *existing praxeologies* and of the development of a logos for web search skills → not only to describe what exist, but identifies or created useful praxeologies

The research techniques are proposed to identify *conditions of access to suitable praxeological* offer and *conditions for their dissemination* - Ecological analysis of Web search practices.

Study case 6: Digital transformation of craftsmanship and craft training (Rivier-Perret, 2020)

RQ: What transformation in training by digital developments in manufacturing processes?

Analysis the tension between technological innovations and traditional manual practices in the transmission of the profession → observation and online questionnaires

The research techniques are propose to analyse the existence of different professional's reference models (different institutions and institutional positions) and training contexts. Weak institutionalisation of professional $\mathscr{P}$.

Herbartian research questions in Line 1

The teaching profession within the ATD

- $RQ_{L1.3}$: What does ATD contribute to the delimitation of the *teaching profession* and the *training for the profession*?
  - Artaud (2005) → TAD comme théorie pour la formation des professeurs.
  - Bosch & Gascón (2017) → Basic assumptions questioned by the ATD.
  - Bueno-Ravel (2007) → Teacher role in the internal didactic transposition.
  - Chevallard (2022) → Challenges and advances of in TE within the ATD.
  - Otaki & Asami-Johanson (2022) → Paradidactic ecosystem.
  - Pressiat (2017) → Apports de la TAD à la profession et leur réception.
  - Rasmussen, Østergaard & Pressiat (2020) → From the depersonalisation of problems to its institutional approach.

Herbartian research questions in Line 1

ATD and other "cultures" of research in the teaching profession

- $RQ_{L1.4}$: What shape the emergence, stabilisation, and transformation of research about professionalization and affect the legitimacy and recognition of research approaches and results?

  (Mizoguchi et al., 2020; Miyakawa & Shinno, 2026)

Problems related to the teaching profession

- $RQ_{L1.5}$: What constitutes a *problem of the teaching profession*, and how can such professional problems be identified and analysed?

  (Cirade, 2006 and 2017; Ruiz-Olarría & Sierra, 2011; Ruiz-Olarría et al., 2014)

III. Line 1: Professions and their ecology

Cirade, G. (2017). Éléments d'écologie des problèmes de la profession. CITAD4

Elements of ℛ praxis ⟺ *Elements of the ℛ logos*

- RQ1: How do difficulties encountered in the *métier* become problems of the profession? – Implementation of a TE device: "questions of the week". Analysis of the diversity of difficulties encountered by pre-service teachers (Cirade, 2006).
- RQ2: What do these difficulties reveal about the 'degree of professionalization' of the teaching profession and its noosphere? – Analysis of the conditions and constraints related to the degree of professionalisation.
- RQ3: How are praxeologies for the profession produced, disseminated, and transformed? – Praxeological analysis of types of praxeologies.

- *Métier* and profession
- The *noosphere* of the métier of teachers
- *Problems of the profession*
- *Mathematics* as a professional problem
- La notion d'*archiécole* (Artaud, 1993) and that of *praxeologies for the profession* (Chevallard & Cirade, 2009)

III. Line 1: Professions and their ecology

Cirade, G. (2006). *Devenir professeur de mathématiques : entre problèmes de la profession et formation en IUFM. Les mathématiques comme problème professionnel*. PhD dissertation.

RQ: What difficulties do you encounter when, as a mathematics student, you decide to become a mathematics teacher?

Implementation of a TE device: "questions of the week". Analysis of the diversity of difficulties encountered by pre-service teachers. Clinical research | clinique des formations

"From the perspective of what is commonly referred to as research methodology, our work falls within a field that is undoubtedly underdeveloped or at least still relatively unknown today, namely clinical training. This, of course, requires training programs to be observed and a system for collecting and processing this information.

In terms of training, we clinically observed the training of trainee mathematics teachers at the IUFM in Aix-Marseille over the two years of the program. For several years, we have been involved as researchers in this training program, with exceptional, virtually unrestricted access to all of its components and archives. This work was facilitated by the practice of the educator[...]. [p.7, our translation]

III. Line 1: Professions and their ecology

Ruiz-Olarría & Sierra (2011). La formación matemático-didáctica del profesorado de secundaria. CITAD3.

Elements of $\mathcal{RP}$praxis

- RQ1: Which professional questions emerged in a different TE programme? Which are systematically silenced or excluded usually in TE? – Analysis of professional questions in a different TE contexts.
- RQ2: What praxeological equipment is considered necessary or useful for the teaching profession? – Analysis of the different praxeologies involved + Identification of gaps between expected and available praxeologies.

Elements of the $\mathcal{RP}$logos

- Distinction between different types of praxeologies:

  Mathematical $\mathscr{P}$ to be taught

  $\cap$

  Mathematical $\mathscr{P}$ for teaching

  $\cap$

  $\mathscr{P}$ for the profession

- Prevalence of monumentalism in TE
- Phenomena of the *silence of infrastructure* in TE

Line 2 — *Praxeological equipment and its fragility*

Line 2 focuses on identifying and analysing the praxeological equipment required for training and for being part of a profession. It also includes the analysis of its fragility and incompleteness.

> Given a professional project Π_0, what praxeological equipment $\mathscr{P}$ exist and what is missing or underdeveloped?

Herbartian research questions in Line 2

Mathematical and didactic praxeologies to be taught and for teaching

- $RQ_{L2.1}$: What $\mathscr{P}$ *to be taught and* $\mathscr{P}$ *for teaching* do exist or may exist to carry out a given teaching activity? Which praxeological needs?

  At the level of specific disciplinary domains, sector or themes:

  Numberal systems and numeracy: Lacasta & Wilhelmi (2007), Larguier & Bronner (2011), Ruiz-Higueras & García (2007); Sierra et al. (2007, 2017) | *Geometry:* Olivero et al. (2017) | *Topology:* Bastán et al. (2007) | *Differential calculus:* Lucas et al. (2020) | *Rational numbers:* Putra & Winsløw (2019) | *Negative numbers:* Cid & Bolea (2010), Cid et al. (2017, 2019) | *Statistics:* Chevallard & Wozniak (2011) | *Probability*: Bourgade et al. (2024) | *Quadratic questions*: Viviano (2011), among others.

  About the interrelation of domains: Bronner & Santos (2017),

  About modelling: García et al., (2007), Florensa et al. (2019)

  About relations to different disciplines: Cissé & Dorier (2007, mathematics and physics).

III. Line 2: Praxeological equipments and its fragility

Putra, Z., & Winsløw, C. (2019). A framework for a comparative study of pre-service elementary teachers' knowledge of rational numbers. CITAD5.

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of the $\mathcal{RP}$ logos*

- RQ1: How can ATD helps to study pre-service elementary teachers' mathematical knowledge about rational numbers? – Selection of hypothetical tasks + construction of a 'praxeologies reference models' and analysis of practice and discourses of teachers.
- RQ2: What similarities and differences can be identified between Danish and Indonesian proposals about rational numbers? – Use of 'praxeologies reference models' for

  - Need of references models to have access to individual and institutional $\mathcal{P}$
  - Difficulties of having access to the mathematical and didactic *logos* of $\mathcal{P}$ - 'muteness' of $\mathcal{P}$

Interventionist techniques: Asking teacher to solve and discuss the tasks, to analyse the emerging $\mathcal{P}$.

T_1: given a positive fraction, $\frac{a}{b}$, determine other fractions that are equal to it.

We can describe some possible mathematical techniques to solve the tasks of type T_1:

τ_{11}: compute correct equal fractions of $\frac{a}{b}$ by multiplying/dividing each numerator and denominator by the same positive integer.

τ_{12}: first represent $\frac{a}{b}$ in a model such as a rectangle or a circle diagram, then c model for $\frac{a}{b}$ by dividing it into 2, 3, or more parts. Finally, it can be shown that generate equal fractions, e.g. as follows:

$\frac{3}{4}$ $\frac{3\cdot2}{4\cdot2}=\frac{6}{8}$ $\frac{3\cdot3}{4\cdot3}=\frac{9}{12}$

τ_{13}: first change $\frac{a}{b}$ into a decimal, then find another fraction that is equal to that dec

technique). Meanwhile, the didactic techniques τ_{21}^*, τ_{22}^*, τ_{23}^* τ_{24}^*, τ_{25}^*, and τ_{26x}^* correspond respectively to τ_{21}, τ_{22}, τ_{23}, τ_{24}, τ_{25}, and τ_{26}^-. There are also other possible didactic techniques that some of them can be variants of those techniques (coded by adding a letter):

τ_{12a}^*: represent both fractions into one (two different) number line(s) and show pupils that both fractions stand in the same point.

τ_{15}^*: present and explain the mathematical task T_1 into an appropriate contextual or real life situation, e.g. a task related to share pizzas or cakes.

τ_{15x}^*: presents inappropriate contextual or real life situation for the mathematical task T_1 or suggest to teach pupils through a contextual or real life situation but do not know how to do that. (adding x to represent an inappropriate didactic technique)

III. Line 2: Praxeological equipments and its fragility

Sierra, T., Bosch, M., & Gascón, J. (2007). Interrelación entre lo matemático y lo didáctico en la reconstrucción escolar de los sistemas de numeración. CITAD1.

Elements of $\mathscr{RP}$ praxis ⟺ *Elements of the $\mathscr{RP}$ logos*

- RQ1: What can be considered the *raison d'être* of MPs, in the case of *number systems* and *continuous magnitudes?* How can it change according to I? – Analysis of possible *raison d'être* and selection of generating question/s Q
  - The MP about number system are totally *naturalise* by students-teachers - 'muteness' of $\mathscr{P}$
- RQ2: Given a question Q that we want to be studied in an educational institution I, how can we design and manage the study process in TE to construct the MP that provides the answer to that question in institution I'? – Construction of an explicit Reference Epistemological Model (REM) around Q: REM about numeral systems
  - Future teachers must be introduced to new *mathematics for teaching*, where they meet how to explain the *raison d'être* and can dynamically manage local OMs in school.
- RQ3: How can this MER help to consider a reference didactic models? – Construction of Reference Didactic Model (RDM) based on the previously delimited REM and including the *raisons d'être* of MP

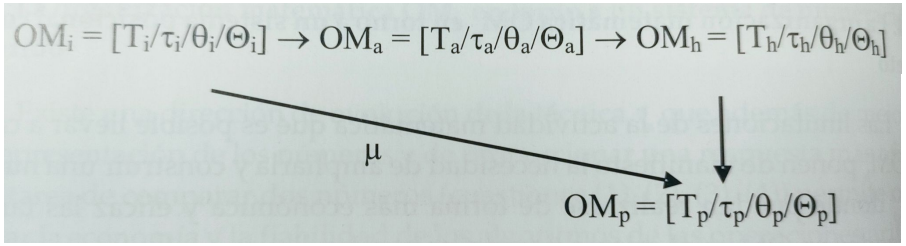
III. Line 2: Praxeological equipments and its fragility

Sierra, T., Bosch, M., & Gascón, J. (2007). Interrelación entre lo matemático y lo didáctico en la reconstrucción escolar de los sistemas de numeración. CITAD1.

RQ: How can natural numbers (NS) be expressed in written form in a way that is useful for developing arithmetic skills at school?

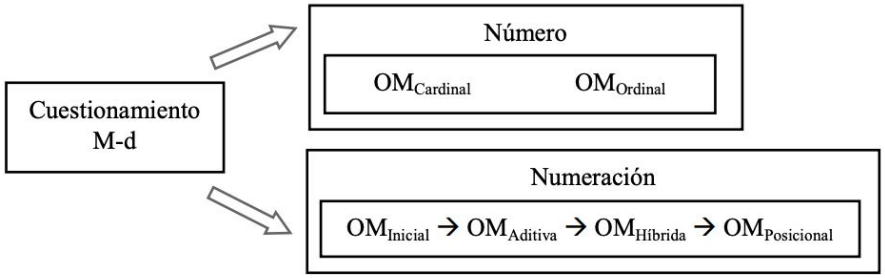
Additive NS

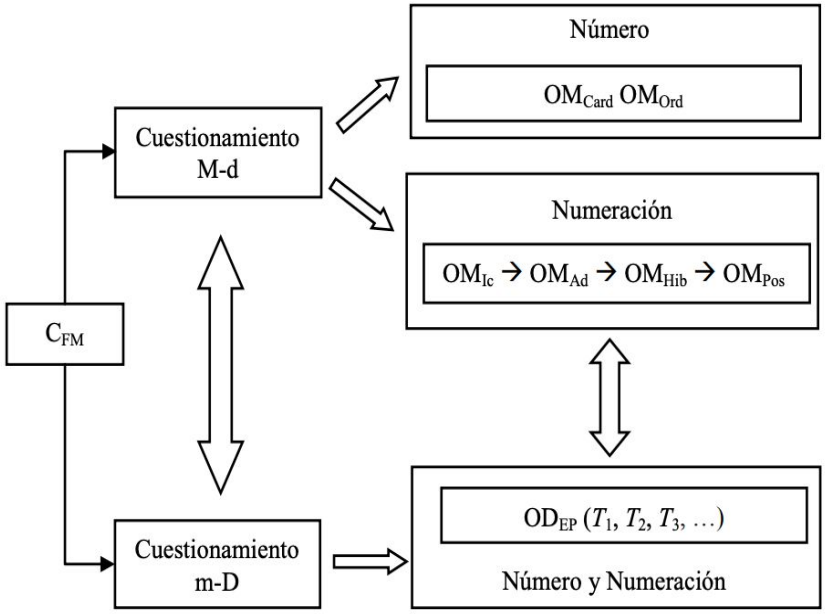
Hybrid NS



Sierra et al. (2007, p. 368)

Place-value NS





Ruiz-Higueras & García (2002, pp. 202–206)

Need to develop a REM that serves as reference for the analysis of the 'spontaneous' and dominant epistemologies and for the development of didactic proposals [RDM]

III. Line 2: Praxeological equipments and its fragility

Florensa, Bosch & Gascón (2019). CITAD 5
Florensa, Bosch & Gascón (2021). Q-A maps as an epistemological tool in teacher education. JMTE.

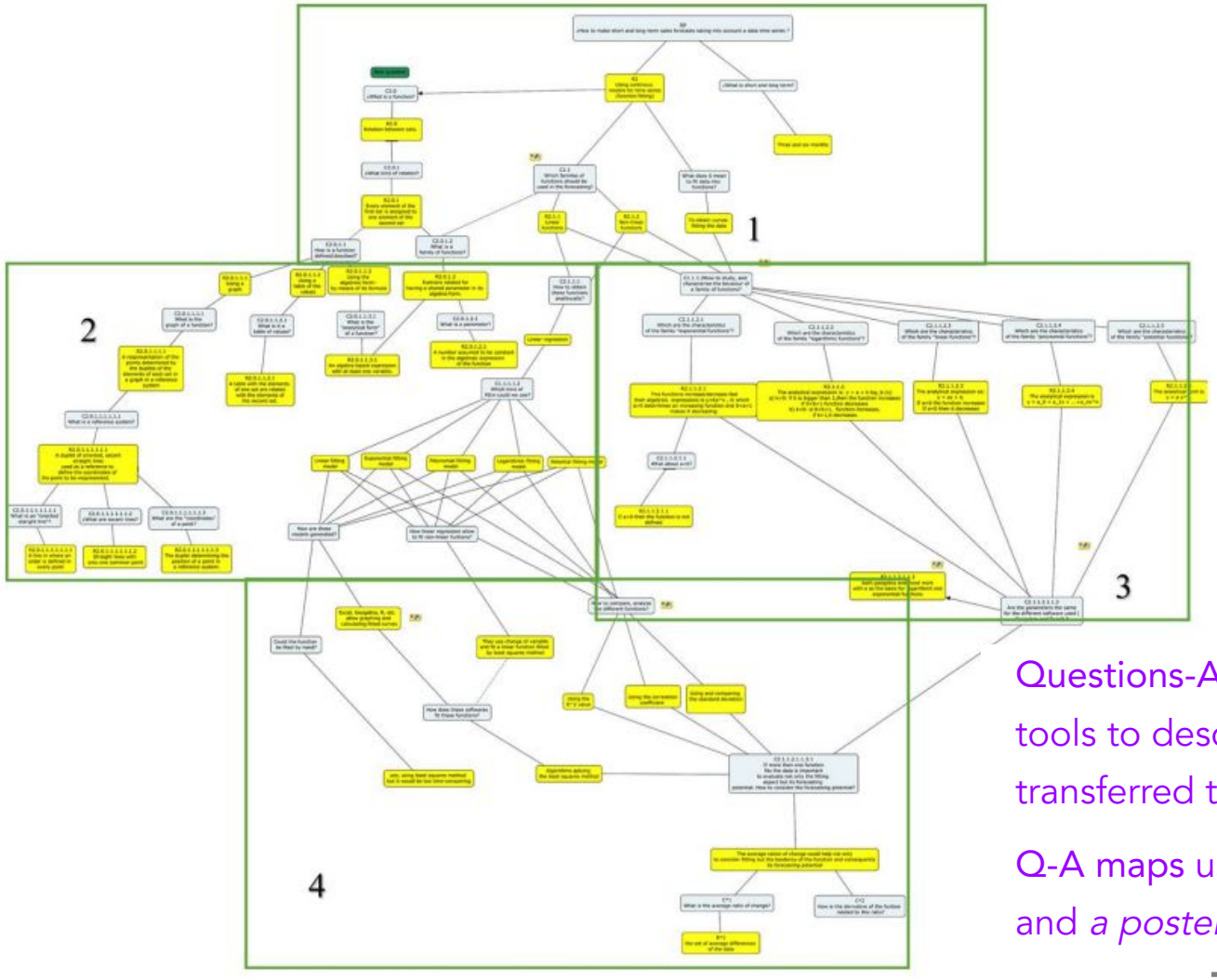


Fig. 3 Example of the Q–A map of one of the groups

Questions-Answers maps appear as tools to describe the REM, also transferred to teacher education.

Q-A maps used in the *a priori*, *in vivo* and *a posteriori* analysis.

Herbartian research questions in Line 2

Praxeologies for the profession and its fragility

- $RQ_{L2.2}$: What *praxeological levels* to distinguish for the teaching profession, what needs came from these different levels? Schneider (2011), Wozniak (2020)
- $RQ_{L2.3}$: What *fragilities* are shown in teachers' praxeological equipments through the analysis of the mathematical and didactic $\mathscr{P}$?

  Tran Luong, Bessot & Dorier (2007, ostensifs, integrals); Asami-Johansson & Attorps (2019, area); Rodríguez-Quintana et al. (2019, numbers in preschool), among others.
- $RQ_{L2.4}$: What *logos* support professional which *praxis* of teachers in certain periods, tendencies and societal-school-... changes? What ecological conditions could support the development of certain didactic $\mathscr{P}$ (such as, project method)? Carrillo & Sánchez (2011), Sánchez & Carrillo (2019), Sánchez et al. (2020)

Specificities on the research techniques: Historical methods analysis, selection of primary resources. Praxeological and Ecological analysis taking into account historical period, institutional conditions and teaching practices.

Herbartian research questions in Line 2

Professional infrastructure supporting teachers praxeological equipment

- $RQ_{L2.5}$: What elements of professional *infrastructure* allow teachers to share, work on and disseminate their praxeological equipment?

  (Miyakawa & Winsløw, 2011, 2013, 2019; Miyakawa, 2022; Mizoguchi et al., 2020)

  (García et al., 2022, 2026; Lendínez, García & Lerma, 2020)

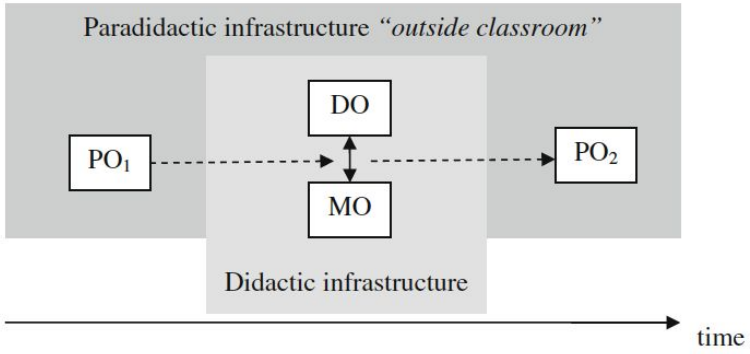
III. Line 2: Praxeological equipments and its fragility

Miyakawa & Winsløw (2011). CITAD3.
Miyakawa & Winsløw (2019). Paradidactic infrastructure for sharing and documenting mathematics teacher knowledge: a case study of "practice research" in Japan. JMTE.

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of the $\mathcal{RP}$ logos*

- RQ1: What elements of the japanese teachers' *infrastructure* allow teachers to access, produce, share and disseminate MP and DP? – Identification of elements of "practice research" as part of japanese teachers' infrastructure + analysis of interrelations
- RQ2: How does 'practice research' (*jissen kenkyū*) become theorised, validated, and published as shared professional knowledge? – Case study of two episodes + analysis of infrastructural conditions through teachers discussion (task design and lesson planning)

- *Paradidactic* infrastructure



- "Lesson study", "Open lesson", "practice research"... as part of paradidactic infrastructure
- Systemic multi-level ecological analysis of how infrastructural elements interact

Line 3 — *Transformation of professional practices*

Line 3 focuses on the transformation of professional practices through the design, implementation and analysis of (new) didactic devices.

Given a professional project Π_0, what conditions might be created so that the required praxeological equipment could be integrated and institutionally sustained to transform professional practices?

Herbartian research questions in Line 3

Development of specific REMs for the profession in a change of paradigm

$RQ_{L3.1}$: How to modify or create (new) infrastructures to support students' learning of **specific topics** in university mathematics?

- Plane analytic geometry (Lobo & Almouloud, 2026)
- Bio-Mathematics (Serrano & Lucas, 2026; Hausberger, 2026)
- Business and administration degree (Serrano et al., 2013)
- Linear Algebra (Hakamata, 2026)
- Additive-multiplicative numeral systems; OMp around place-value numeral systems (Sierra & Gascón, 2018)
- Interrelations between numerical-algebraic and geometric mathematical domain (Bronner & Santos Farias, 2007)
- …

Herbartian research questions in Line 3

Development of didactic devices to smooth Klein's discontinuities

$RQ_{L3.2}$: What kind of teaching devices can be useful to smooth Klein's discontinuities in the transition to the profession or to the training for it?

- Capstone courses
  - Calculus (Winsløw & Grønbæk , 2014)
  - Real numbers (Barquero & Winsløw, 2023; Huo, 2024)
  - Local and global aspects (Winsløw 2023, 2025)
- Bridging courses (Serrano, 2013; Serrano, Bosch & Gascón, 2020)

III. Line 3: Transformation of professional practices

Winsløw & Grønbæk (2014). Klein's double discontinuity revisited: contemporary challenges for universities preparing to teachers to teach calculus. RDM.

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of $\mathcal{RP}$ logos*

- RQ1: How to revisit Klein's double discontinuity in ATD-notions? - Reformulation of Klein's problem formulation and proposal in view of

$$R_{HS}(s,o) \rightarrow R_U(\sigma,\omega) \rightarrow R_{HS}(t,o)$$

- RQ2: What are the main obstacles to building these relationships? - Observations of student's work on concrete tasks on the basis of praxeological analyses.
- RQ3: What further developments $R_U^*(\sigma,\omega)$ may be desirable? - Formulating first answers on the basis of identified obstacles answering RQ2.

- ATD modelling of cognition
- *institutional relativity* of praxeologies
- *praxeological modeling* of tasks

III. Line 3: Transformation of professional practices

Serrano, Bosch & Gascón (2020). An overview of "bridging courses" from the ATD perspective. CITAD6

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of $\mathcal{RP}$ logos*

- RQ1: How to evaluate the infrastructure provided to students by existing bridging courses? - Studying the didactical moments in terms of praxeologies: courses favor the isolation and disarticulation of the mathematical praxeologies studied in secondary schools. (Fonseca, 2004)
  - *Didactical moments* of study
  - *praxeology*
  - *local* praxeology
  - *didactic contract*
- RQ2: How to design a course to overcome those discontinuities? - Identifying initial questions for implementing research and studies courses making possible to integrate specific praxeologies into a relatively complete local praxeology.

Q_0: Determine for what sales company's incomes are greater than costs (or unit prices higher than average costs, or profits greater than a given value, or costs less than a given value, etc.).

Herbartian research questions in Line 3

Creating devices to support developing new infrastructures

- $RQ_{L3.3}$: How to modify or create (new) infrastructures to supporting the necessary praxeological equipment for inquiry oriented teaching? Through which proposals?

SRP for Teacher Education (SRP-TE) or other devices

Preschool TE: Lerma et al. (2024)

Primary school TE: Ruiz-Olarría, Sierra & Gascón (2014), Ruiz-Olarría & Sierra (2017)

Secondary school TE: Barbosa & Bittar (2026), Barquero, Bosch, & Romo-Vázquez (2018, 2019), Florensa, Bosch & Gacón (2019), Rasmussen (2016), among others.

SRP-TE at different levels: Barquero, Florensa & Ruiz-Olarría (2022)

III. Line 3: Transformation of professional practices

Barquero, Ruiz-Olarría & Florensa (2022). The education of school and university teachers within the paradigm of questioning the world. Routledge.

Elements of ℛ𝒫 praxis ⟺ *Elements of the ℛ𝒫 logos*

- RQ1: What teacher professional questions have been used as generating questions and how do they evolve during the development of SRP-TE? – Identification of generating questions and analysis of common questioning among experiences
- RQ2: What ATD epistemological and didactic tools have been transferred by teacher-students in the experiences? – Analysis of the use and function of specific tools

- Generating professional questions
- SRP-TE with previously developed SRP or with SRP design
- Didactic Engineering as common methodology for the design of SRP-TE
- Adaptation of tools for TE in the *a priori*, *in vivo* and *a posteriori* phases:
  - Q-A maps
  - Lesson plans including QA maps + chronogenesis, mesogenesis, topogenesis aspects
  - Ecological analysis

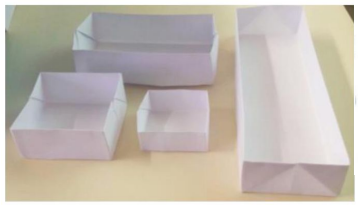
III. Line 3: Transformation of professional practices

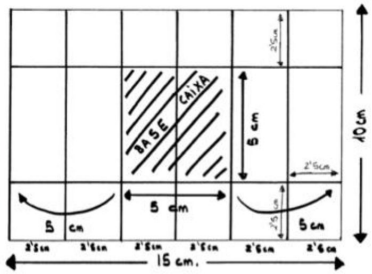
Barquero, Ruiz-Olarría & Florensa (2022). The education of school and university teachers within the paradigm of questioning the world. Routledge.

Case 1: Cake box SRP-TE for pre-service Primary school teachers

→ How to integrate mathematical modelling in primary school education?

→ Experience (Module 1) and analysis (Module 2) of the modelling process: from numerical-measurement models to pre- and algebraic models.



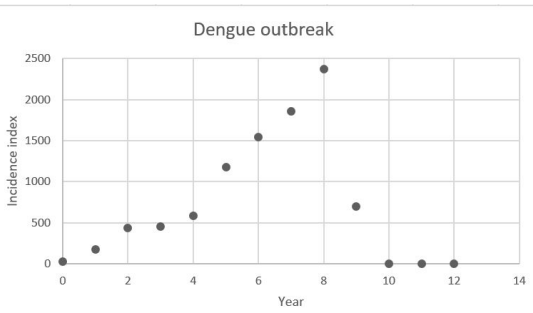


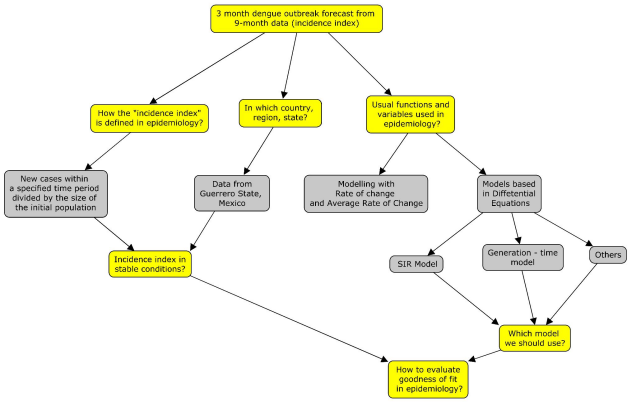
Case 3: SRP-TE for engineering lecturers

→ What is the dominant approach to the teaching of data treatment, functions,...?

→ Questioning the dominant epistemology at university level. Modelling as a way to re-define the knowledge to be taught.

Year	Incidence
0	31
1	179
2	438
3	454
4	587
5	1176
6	1543
7	1859
8	2373
9	696
10	0,1
11	0,05
12	0





Herbartian research questions in Line 3

Conditions and constraints for creating devices for PQW

- $RQ_{L3.4}$: How and which institutional conditions and constraints can be identified modifying and creating infrastructures for inquiry oriented teaching (in PQW)?

  (Artaud, 2017, 2026; Bašić & Milin Šipuš, 2024; Bourgade, 2024; Chevallard, 2004; Cirade & Crumière, 2022; García, 2017; Hochmuth & Peters, 2024; Jessen & Rasmussen, 2020; Jessen, 2022; Júnior & Fernandes, 2026; Kuzuoka & Miyakawa, 2020; Otero et al., 2013, 2020; Parra et al., 2013; Strømskag, 2024; Verbisck, Barquero, Bittar & Bosch, 2026; among others)

III. Line 3: Transformation of professional practices

Artaud (2017). Enquêter pour questionner le monde: conditions et infrastructures didactiques. CITAD 4.

Elements of $\mathcal{RP}$ praxis

- RQ1: Which teaching infrastructures are required in the process of implementing a paradigm of questioning the world?
  - Discussing and reconstructing infrastructure on the basis of observations and spiral models developed in the project PRIMAS: Q^{δ} "How can we investigate the professional world of Primas to develop their praxeological skills?" (García, 2017)

Professional development activity	Environment for *study M*	Investigative actions
(A) Analysis of the type of questions and their function	Teachers; Teachers' responses $(R^{\diamond}_{i})_x$ = types of questions they use and the role they play; Groups; discussion	Observation and analysis
(B) Effective questions to support *inquiry-based learning* (IBL)	Teachers' answers $(R^{\diamond}_{i})_x$ = types of questions that could be effective; Trainer: $(R^{\diamond}_{i})_y$ = five principles for effective questioning; Groups; discussion	Observation, analysis and evaluation
(C) Observation and analysis of a classroom session	$(R^{\diamond}_{i})_x$ and $(R^{\diamond}_{i})_y$ are available Video of a class; Groups; discussion	Observation, analysis and evaluation
(D) Design and implementation of a classroom session	Trainer; $(R^{\diamond}_{i})'_y$ = guide for developing a plan of effective questions; Teacher *X's* class (implementation)	Development of $R^{\diamond}_{i}$ for Establish an $R^{\bullet}$; defence and dissemination

III. Line 3: Transformation of professional practices

Artaud (2017). Enquêter pour questionner le monde: conditions et infrastructures didactiques. CITAD 4.

Further elements of $\mathcal{RP}$ praxis ⟺ *Elements of $\mathcal{RP}$ logos*

- RQ2: Which conditions are necessary for this type of infrastructure to exist? - Highlighting and explicating (partly implicite) conditions in steps of study processes related to RQ1, e.g. teachers are supposed to know how to observe practice.
- RQ3: What are differences between these conditions and the requirements of a PVW? - Reformulations related to, e.g., needs of works for evaluation,, in terms of moments and gestures of study and their dialectics: "sufficiently rich" environments for study and adequate realizing the moment of institutionalization.

- *Herbatian scheme*
- *naturalisation* of observation
- types of *evaluation* of responses
- *media* and *milieu* dialectics
- *infrastructure* and *conditions*
- modelling in terms of *study moments*

III. Line 3: Transformation of professional practices

Jessen (2022). Study and Research Paths, Ecology and In-service Teachers. IRP

Elements of ℛ𝒫 praxis ⟺ *Elements of ℛ𝒫 logos*

- RQ1: Which parts of the education system impose constraints and conditions on the implementation of SRP-based teaching? – Interpreting students' doings and explanations (e.g., backwash effect of high-stake written exit examinations), study programs (e.g., teachers' lack of experience with interdisciplinary teaching) and ministerial guidelines (e.g., "competencies").
- RQ2: How can we introduce the teachers to SRP and ATD without using the full theoretical framework? - Adapting in ATD developed didactic devices, e.g., using knowledge-maps.

- scale of level of *codeterminacy*
- dialectics between infrastructure and superstructure
- dialectics of institution, position and person
- SRP-TE, SRA
- QA-maps
- media-milieu dialectics

Line 4 — ATD in dialogue with other theoretical frameworks

Line 4 operates at a meta-theoretical level by analysing how problematic questions about the profession conceptualised—or left implicit—in other frameworks, and by showing the specific reformulation and contribution of the ATD.

> Line 4 = Meta-analysis of {P1, P2, P3} across theoretical frameworks and how ATD approach the RQ addressed

Herbartian research questions in Line 4

Research praxelogies with explicit involvement of other approaches

$RQ_{L4.1}$: Which research approaches are considered? In which ways and with which specific aims the theoretical frameworks 'collaborate'?

- TDS and ATD

  Bosch & Gascón (2017), Chaachoua, Bessot, Romo & Castela (2020), Mercier (2007), Ruiz-Higueras & García (2007), Winslow (2017), Wozniak & Margolinas (2022), Otaki (2024), among other papers.

- Metadidactic transposition (Otaki, 2024; Florensa, Bosch & Gascón, 2021)
- Mathematical Knowledge for Teaching (MKT) (e.g., MCK, PCK) (Ruiz-Olarría, Bosch & Gascón, 2017)
- General pedagogical theories, e.g., theories about theory-practice relation (Østergaard, 2017)

III. Line 4: ATD in dialogue with other theoretical frameworks

Bosch, & Gascón (2017). Asunciones básicas de la cultura didáctica cuestionada por la TAD. El problema de Klein y la formación del profesorado. CITAD 4

- RQ: What basic principles of the **dominant didactic culture** shape the way the teaching profession and teacher education is proposed? What **'transformative' assumptions** are shared by the TDS and ATD ('Brousseaunian' revolution)

Making explicit a *logos* to analyse dominant models in the teaching profession and TE, and to justify ATD/TDS-based proposals

- Principle 1: 'Doing mathematics' and 'teaching mathematics' are (relatively) independent activities.
- Principle 2: The dominant epistemological models in institutions (school and academic) are not questionable by didactic research.
- Principle 3: Didactic problems are, first and foremost, individual problems and, to a large extent, cognitive problems.
- Principle 4: Didactic problems can be addressed by studying micro-didactic phenomena that refer to specific content to be taught.
- Principle 5: The classroom is the ideal setting for extracting the empirical data needed to study didactic problems.

Our translation from Bosch & Gascón (2017)

III. Line 4: ATD in dialogue with other theoretical frameworks

Wozniak & Margolinas (2022). Introduction of Ordinal Number at the Beginning of the French Curriculum: A Study of Professional Teaching Problem. IRP.

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of the $\mathcal{RP}$ logos*

- RQ1: How can the main characteristics of the number as position concept be transposed into situations (Brousseau, 1998)? – Experimental choices are based on, e.g., analyses of curricula, milieu (e.g. school books), situation's stake, epistemological bases of notions.
- RQ2: How to identify teacher's mathematical and didactical knowledge needs? - Organizing three types of observations (research engineering) and praxeological analyses of observations regarding teachers' practices (ostensives, techniques …), effects of logos (mute, weak, strong), degree of conformity (REM) (Chevallard, 2020)

- didactic transposition
- praxeological analysis
- didactic engineering design
- fundamental situation
- action and formulation
- milieu
- effect of logos
- degree of conformity

III. Line 4: ATD in dialogue with other theoretical frameworks

Ruiz-Higueras & García (2007). Análisis de las praxeologías didácticas: implicaciones en la formación de maestros. CITAD3 + Ruiz-Higueras & García (2011, RELIME)

Elements of $\mathcal{RP}$ praxis

- RQ1: What didactic praxeologies are mobilised by teachers? How to analyse these praxeologies? – Analysis of didactic praxeologies in particular episodes and selected teacher(s)
- RQ2: How to describe the didactic *praxis* and didactic *logos*? – Analysis of the praxis: mathematical and didactic $\mathcal{P}$, reinforced by cronogenetic, mesogenetic and topogenetic didactic techniques and the *logos*: levels of teacher activity (Margolinas et al., 2005)

Nivel +3	Valores y concepciones sobre la enseñanza y el aprendizaje	Es el nivel más general (nivel «noosferiano» o ideológico): reflexión muy general del profesor sobre la enseñanza y el aprendizaje
Nivel +2	Proyecto didáctico global	Concepción general sobre cómo organizar un tema de enseñanza: nociones a estudiar y conocimiento a adquirir
Nivel +1	Proyecto didáctico local	Proyecto didáctico específico de una lección o conjunto de lecciones: objetivos y organización del trabajo
Nivel 0	Acción didáctica	Interacción con los alumnos, decisiones durante la acción
Nivel –1	Observación de la acción de los estudiantes	Percepción de la actividad de los estudiantes y regulación de su trabajo

EPISODIO 4

Ana: Cuando ya se murieron todas las mariposas, decidimos hacer murales gráficos de los diferentes estados de nuestra colección.

La maestra eligió cuatro momentos significativos:
Cuando solo teníamos gusanos.
Cuando comenzaron a hacer crisálidas.
Cuando solo teníamos crisálidas.
Cuando solo teníamos mariposas, incluidas las muertas que guardábamos para mirarlas después en el microscopio.

En su elaboración han intervenido todos los niños pero no todos a la vez, es decir, que han ido pasando por la actividad en grupos de seis o siete, de modo que el mural finalizaba cuando todos los niños de la clase habían aportado su trabajo al mismo.

Lo más interesante de este trabajo es que, cuando un grupo se iba porque acaba su tiempo y llegaba otro, el primer objetivo de este era ver en qué estado se encontraba la tarea, es decir, contar cuántos elementos teníamos ya dibujados, recortados y pegados y cuántos nos quedaban por hacer, por lo que las tablas en las que íbamos anotando las variaciones de la colección eran imprescindibles, una auténtica herramienta de trabajo.

TAREAS DIDÁCTICAS	TÉCNICAS DIDÁCTICAS
Realizar los momentos del primer encuentro, exploratorio y de trabajo de la técnica (OM_4).	**Técnicas mesogenéticas:** Construir un nuevo medio, relanzar nuevas cuestiones: una vez que ha desaparecido totalmente el sistema, la maestra tiene que construir un nuevo medio para permitir que siga avanzando la actividad matemática de los niños.
Organizar y regular el medio.	Las tablas, que hasta ahora han sido herramientas, se convierten en un auténtico medio a partir del cual la maestra introduce un nuevo problema: reconstruir el sistema en diferentes estados.
Llevar a cabo una nueva devolución del problema a los alumnos.	Integrar organizaciones matemáticas: la maestra articula entre sí OM_1, OM_2, OM_3 constituyendo una nueva praxeología OM_4 que amplía e integra entre sí a las anteriores.
Dotar de una nueva funcionalidad (razón de ser) a objetos matemáticos previamente construidos.	Reforzar el proceso de devolución: la maestra formula necesariamente nuevas cuestiones para que los niños aborden la tarea de producir colecciones y trata de asegurar la devolución del problema, aún cuando el sistema ya no existe.
Institucionalizar la tabla de doble entrada como modelo del sistema.	Gestión de ostensivos: la maestra debe gestionar el ostensivo tabla junto con los ostensivos banda numérica y almanaque.

III. Line 4: ATD in dialogue with other theoretical frameworks

Florensa, Bosch & Gascón (2021). Question–answer maps as an epistemological tool in teacher education. JMTE

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of the $\mathcal{RP}$ logos*

- RQ1: To what extent act QA-maps as *boundary objects*, i.e. that is, become external objects for the community of teachers internal by the end of the course? – Qualitative analyses of the creation and evolvement of Q-A maps and teachers' surveys (Barquero et al., 2013; Jessen, 2014)
- RQ2: To what extent does the use of QA-maps enable teachers to be aware of most important l constraints affecting the design of new teaching proposals in a specific school institution?– Contrasting the Q-A maps with schoolbooks and analysing the media-milieu dialectic in detail.

- meta-didactic transposition
- boundary objects
- mathematical knowledge for teaching (MKT)
- SRP-TE
- reference epistemological model
- prevailing epistemology
- *media-milieu* dialectic

Herbartian research questions in Line 4

Specific research results considering higher levels of codeterminacy

- RQ: How specific *research praxeologies* and results related to higher levels of codeterminacy are integrated within ATD-research?
  - praxeological analyses of mathematics specific to professions (Hochmuth & Peters, 2020, 2022)
  - analyses of didactic praxeologies of teacher students (Hochmuth & Peters, 2024, 2026)
  - specificities in workplaces (already addressed in line 1) (Ladage, 2026)

III. Line 4: ATD in dialogue with other theoretical frameworks

Hochmuth & Peters (2022). About Two Epistemological Related Aspects in Mathematical Practices of Empirical Sciences (& papers about signal theory, e.g. IJRUME 2021).

Elements of $\mathcal{RP}$ praxis ⟺ *Elements of the $\mathcal{RP}$ logos*

- RQ1: How are differences in praxeologies regarding institutions interpreted and explained w.r.t. higher levels of codeterminacy? – Reconstructing the dialectics between practices and needs based on historical and epistemological studies of their constitution, e.g. regarding "quantities" and "measurement"
- RQ2: How do relate praxeological "ideal-types" to individual or institutional practices ? - Reconstructing the state of the art about "describing" vs "explaining" in ideal-types, e.g., Schwemmer's theory of rational explanation

- scale of levels of codeterminacy
- institutional relativity of praxeology
- the notion "discourse"
- ATD-notions as categories to (re-)formulate practices, questions and research done in other disciplines

IV. Concluding remarks and future developments

- Lines 1 – 4 are not independent

  → Considering them separately has been a methodological choice

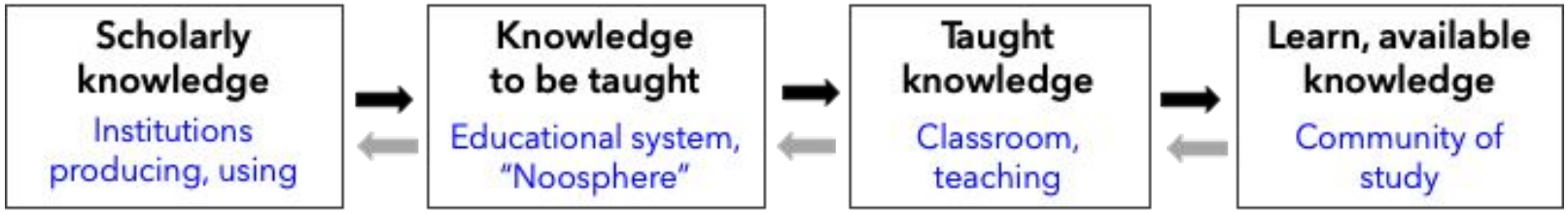
- This decisions have facilitated:
  - Delimit main research topics in each line
  - Propose the 'Herbartian' research questions $RQ_{Li.j}$
  - Identify related papers contributing and select some of them
  - Analyse more in-depth the research praxeologies
    - Describe elements of the *praxis* and of the *logos*, which can help to make explicit the know-how and know-why of or research.
    - As well as think on new research RQ and developments of the research praxeologies.
- All is open to discussion with the ATD-community: lines, topics, reformulation of the Herbartian research questions, etc.

IV. Concluding remarks and future developments

Line 1 — *Professions and their ecology*	
Professions beyond the teaching profession	$RQ_{L1.1}$ $RQ_{L1.2}$
The teaching profession within the ATD	$RQ_{L1.3}$
ATD and other "cultures" of research in the teaching profession	$RQ_{L1.4}$
Problems related to the teaching profession	$RQ_{L1.5}$
Line 2 — *Praxeological equipment and its fragility*	
Mathematical and didactic praxeologies to be taught and for teaching	$RQ_{L2.1}$
Praxeologies for the profession and its fragility	$RQ_{L2.2}$- $RQ_{L2.4}$
Professional infrastructure supporting teachers praxeological equipment	$RQ_{L2.5}$
Naturalistic techniques | Interventionist techniques Inventory and questioning of reading, witness techniques Transpositive analysis, praxeological analysis, reference epistemological/praxeol. models different levels of praxeologies, economical analysis, ecological analysis	

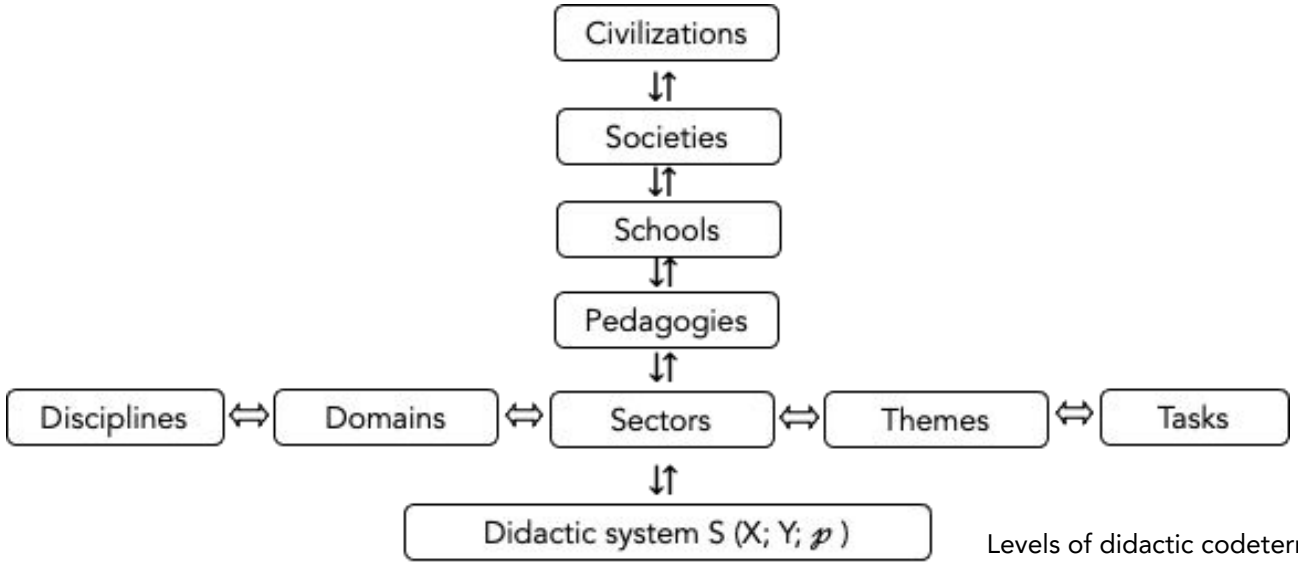
IV. Concluding remarks and future developments

Didactic transposition analysis ← → RQ, Unit of analysis = { I_j , empirical data}



Didactic transposition process Chevallard (1985)

Ecological analysis ← → Unit of analysis | Where are constraints detected, where do they came from?



Levels of didactic codeterminacy Chevallard (2002)

IV. Concluding remarks and future developments

Line 3 — *Transformation of professional practices*	
Development of REMs for the profession in a change of paradigm	$RQ_{L3.1}$
Development of didactic devices to smooth Klein's discontinuities	$RQ_{L3.2}$
Creating devices to support developing new infrastructures	$RQ_{L3.3}$
Conditions and constraints for creating devices for PQW	$RQ_{L3.4}$
Interventionist techniques, Clinical research Didactic Engineering as research methodology: preliminary analysis, *a priori* analysis, *in vivo* analysis, *a posteriori* analysis	
Line 4 — *ATD in dialogue with other theoretical framework*	
Research praxelogies with explicit involvement of other approaches	$RQ_{L4.1}$
Other specific research considering higher levels of codeterminacy	$RQ_{L4.2}$
Research praxeologies themselves	

IV. Concluding remarks and future developments

Interventionist approach ← → Didactic Engineering as research methodology

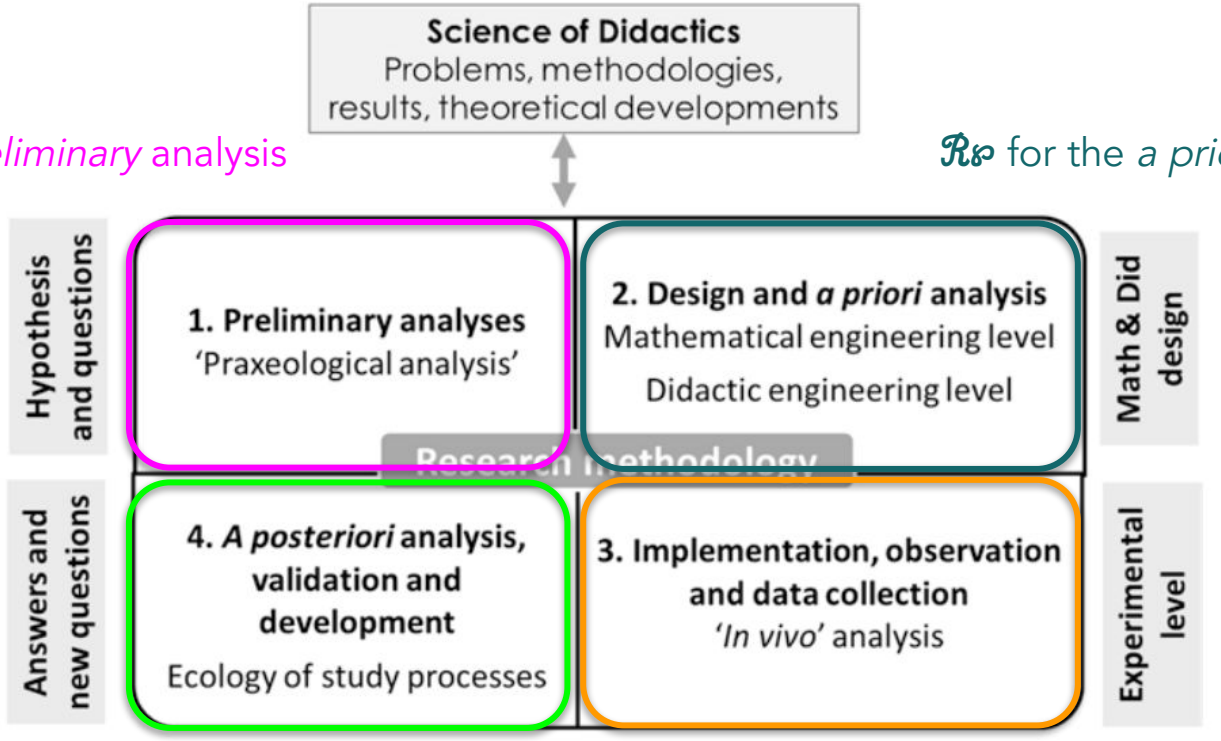


Fig. 8.5 Phases of the DE research methodology within the ATD

Barquero & Bosch (2015, p. 263)

IV. Concluding remarks and future developments

La « méthodologie » concerne donc ce que je nommerai les praxéologies de recherche mises en jeu dans un domaine donné [...]. Je profite de cette remarque pour souligner que le travail collectif sur les praxéologies de recherche en didactique, c'est-à-dire sur "the methods and procedures by which such knowledge and understanding are achieved" et sur "the formulation of systematic and logically coherent methods for the search for knowledge", me semble aujourd'hui plus nécessaire que jamais pour combattre les effets de la routinisation, voire de la quasi-naturalisation des méthodes usitées. (Chevallard, 2011, p. 11)

IV. Concluding remarks and future developments

Necessity to continue in this collective work on the **research praxeologies**…

- To break the **transparency** and **routinisation** of methods we use.
- Understanding the **praxeological-relativity** of different elements.
- Provide a more **specific terminology** to refer to the different elements of the research praxeologies to help on its dissemination.

But, how to enrich the 'static' view of what research consist of? How to describe the dynamicity of the research processes?

$$[S(X_{\mathcal{R}}, Y_{\mathcal{R}}, \mathcal{R}Q_0) \rightharpoonup \{\mathcal{R}\wp_i^{\diamond}, O_j, Q_n, D_p = \{UA_p\}] \hookrightarrow \mathcal{R}A^{\bullet}$$

Thank you very much!

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CITAD 8. CRM, Bellaterra. January 22nd, 2026.